Asymptotic Complexity Sorting

CS143: lecture 11

Konstantinos Ameranis, July 16

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- Except the last input where Algorithm A finishes much much faster

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- The last input is actually a special case where Algorithm A bypasses most of the work needed

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  - For different sizes which one is better changes
- For small inputs all algorithms will take similar times
- But for large inputs a linear and a quadratic algorithm will have very different run times

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- Therefore as $n \to \infty$ $g(n)$ grows at least as fast as $f(n)$
- From this point on we will say that $f(n) \in O\left(g(n)\right)$

Asymptotic Complexity

Symbol	Limit definition	Constant definition	Relation	Name
$f(n) \in O\left(g(n)\right)$	$\limsup_{n\to\infty} \frac{f(n)}{g(n)} \in \mathbb{R}$	$\exists\ n_0 \in \mathbb{N}, c \in \mathbb{R} : \forall\ n > n_0 \quad f(n) \leq c \cdot g(n)$	$\leq$	Big-O
$f(n) \in \Omega\left(g(n)\right)$	$\limsup_{n\to\infty} \frac{f(n)}{g(n)} > 0$	$\exists\ n_0 \in \mathbb{N}, c \in \mathbb{R} : \forall\ n > n_0 \quad f(n) \geq c \cdot g(n)$	$\geq$	Big-Omega
$f(n) \in \Theta\left(g(n)\right)$	$0 < \limsup_{n\to\infty} \frac{f(n)}{g(n)} \in \mathbb{R}$	$\exists\ n_0 \in \mathbb{N}, c_1, c_2 \in \mathbb{R} : \forall\ n > n_0 \quad c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$	$=$	Theta
$f(n) \in o\left(g(n)\right)$	$\limsup_{n\to\infty} \frac{f(n)}{g(n)} = 0$	$\forall c \in \mathbb{R} \exists\ n_0 \in \mathbb{N} : \forall\ n > n_0 \quad f(n) \leq c \cdot g(n)$	$<$	little-o
$f(n) \in \omega\left(g(n)\right)$	$\limsup_{n\to\infty} \frac{f(n)}{g(n)} = +\infty$	$\forall c \in \mathbb{R} \exists\ n_0 \in \mathbb{N} : \forall\ n > n_0 \quad f(n) \geq c \cdot g(n)$	$<$	little-omega

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- As $n \to \infty$ both $f(n)$ and $g(n)$ go to ∞
- In this case L'Hôpital's rule applies
- $$\limsup_{n\to\infty} \frac{f(n)}{g(n)} = \limsup_{n\to\infty} \frac{f'(n)}{g'(n)}$$
- For example: $$\limsup_{n\to\infty} \frac{3+80n}{3+6n+6n^2} = \limsup_{n\to\infty} \frac{80}{6+12n} = 0$$

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$$\lim_{n\to\infty} \frac{a_k n^k + \ldots + a_0}{n^k} = \lim_{n\to\infty} \frac{a_k n^k}{n^k} + \ldots + \frac{a_0}{n^k} = a_k + \ldots + 0 = a_k \in \mathbb{R}$$

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- Product: If $f_1(n) = O\left(g_1(n)\right)$ and $f_2(n) = O\left(g_2(n)\right)$ then $f_1(n) \cdot f_2(n) = O\left(g_1(n) \cdot g_2(n)\right)$

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- Sum: If $f_1(n) = O\left(g(n)\right)$ and $f_2(n) = O\left(g(n)\right)$ then $f_1(n) + f_2(n) = O\left(g(n)\right)$

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Sorting

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Putting things in order

- What do we have:
  - A *list* of *n* elements
  - A comparison function: $\leq$
- What do we want:
  - The *list* has all the same elements as it started with
  - If $i \leq j$, `list[i]` $\leq$ `list[j]`

Sorting

Example

Sorting

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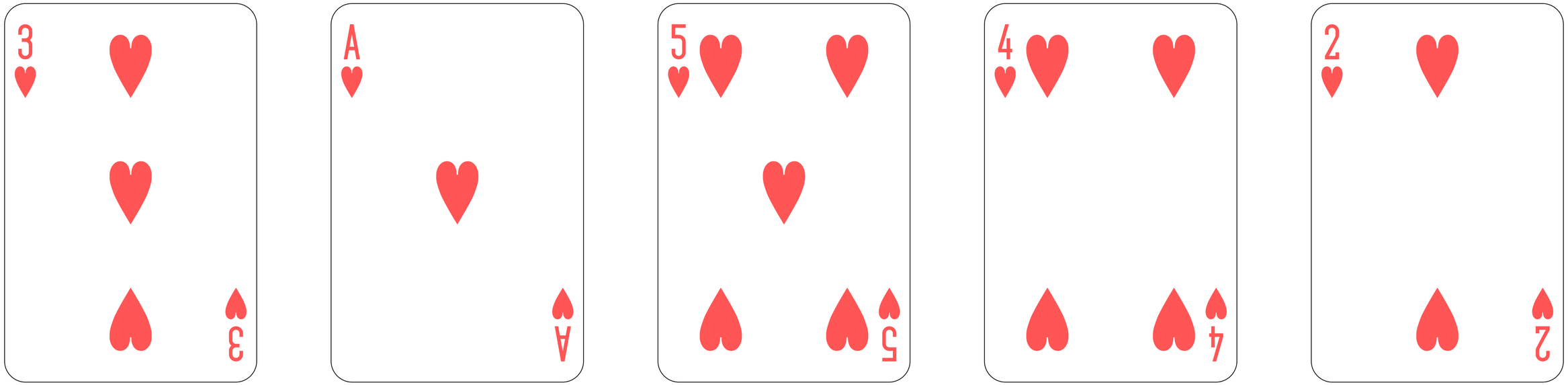
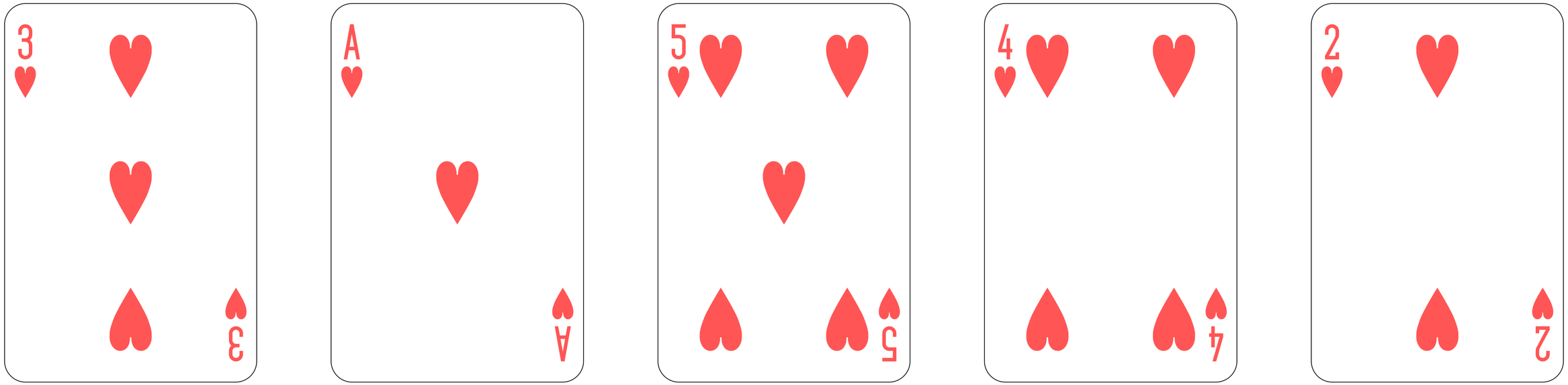


Sorting

Example 1 (how I do it)

- Pick the smallest one and move it to the front



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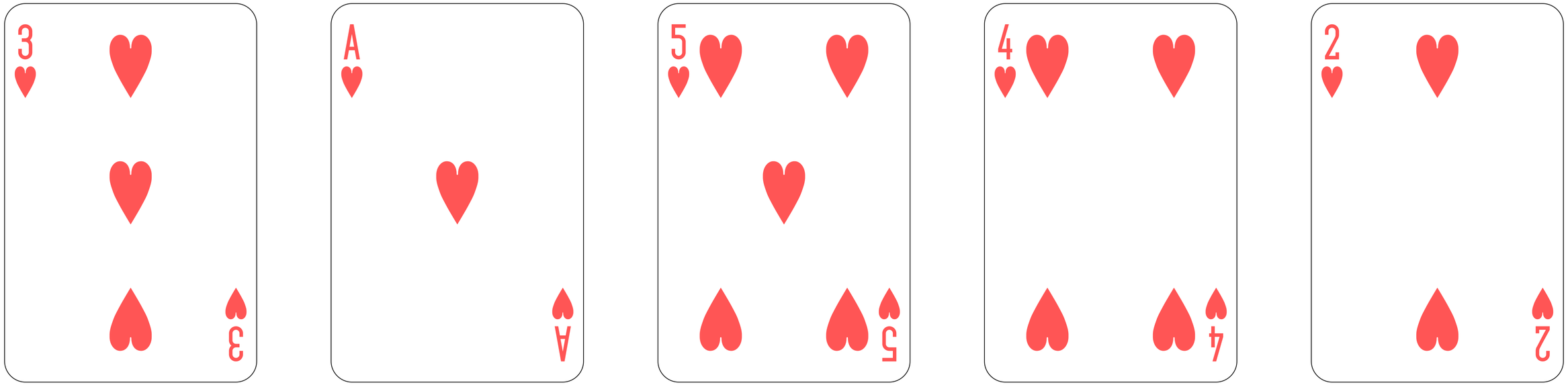

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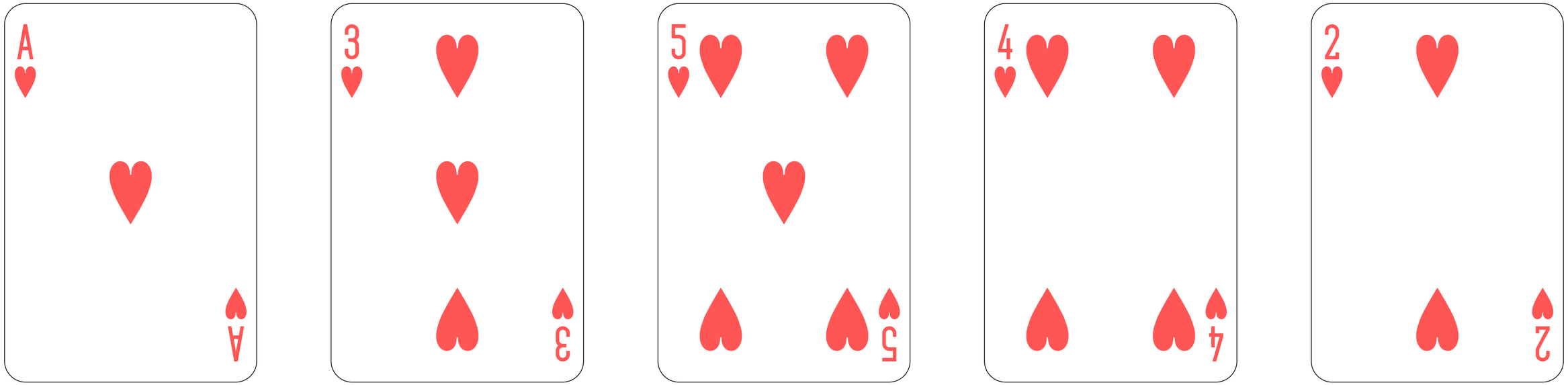

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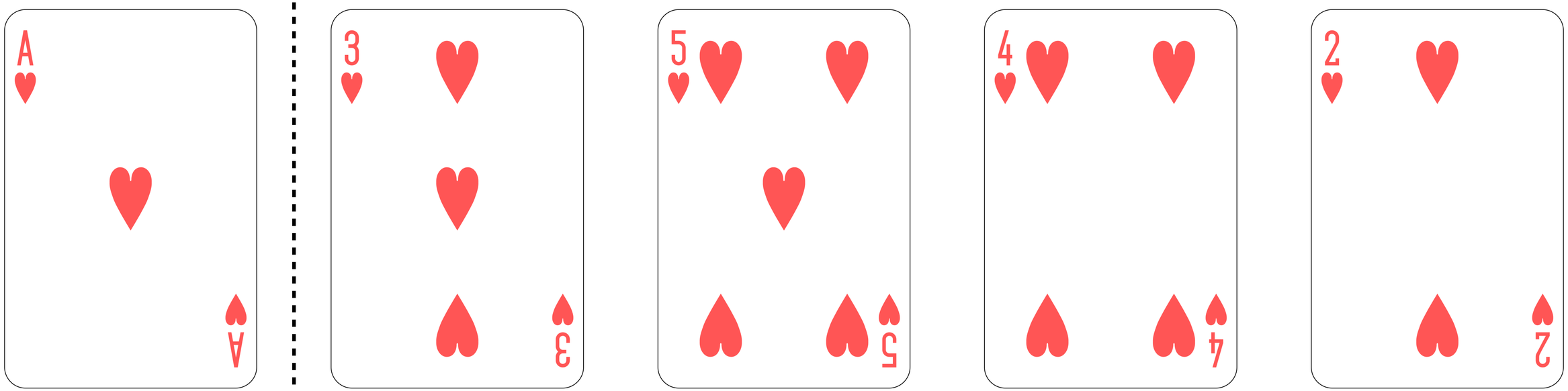

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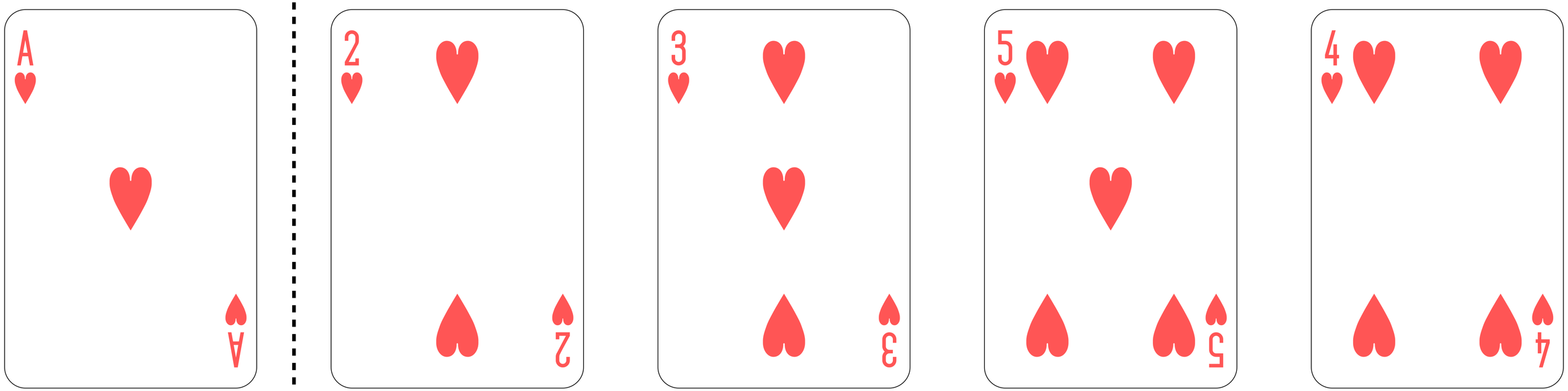

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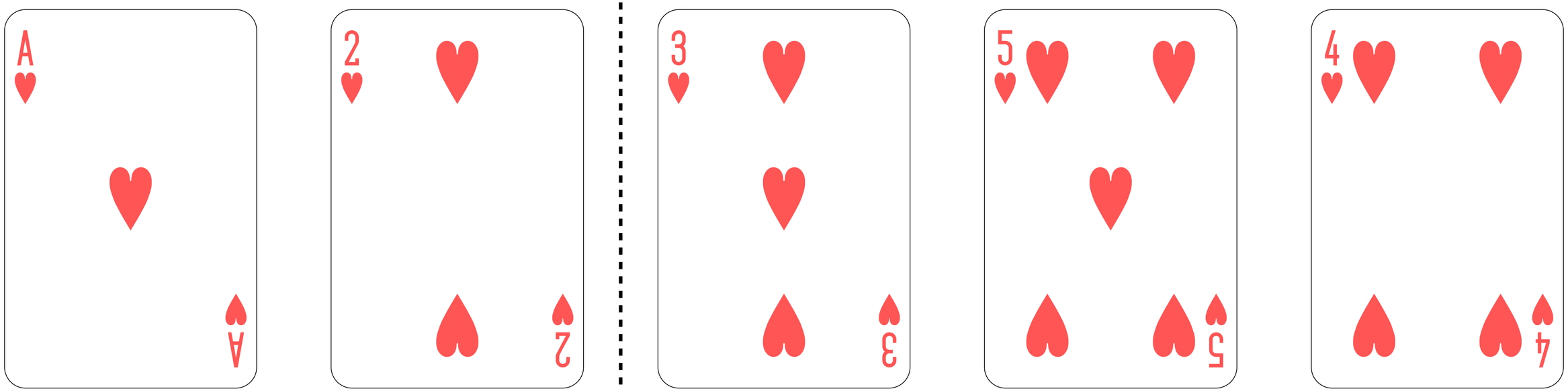

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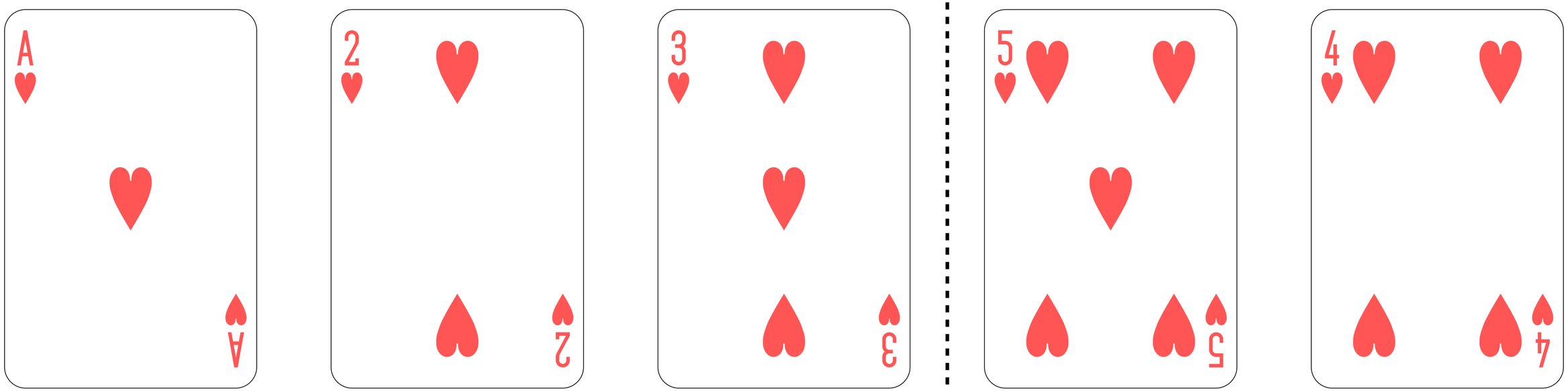

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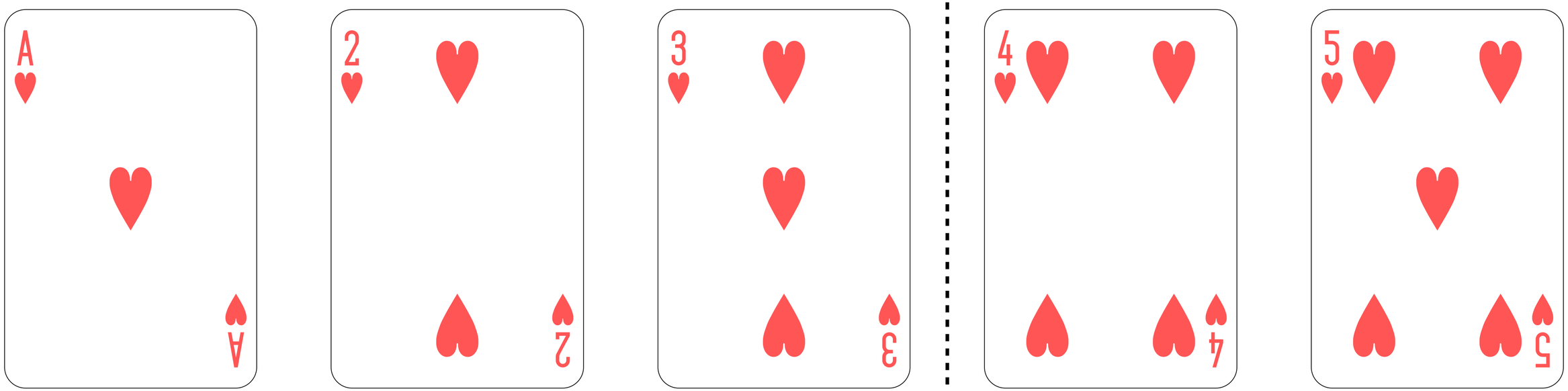

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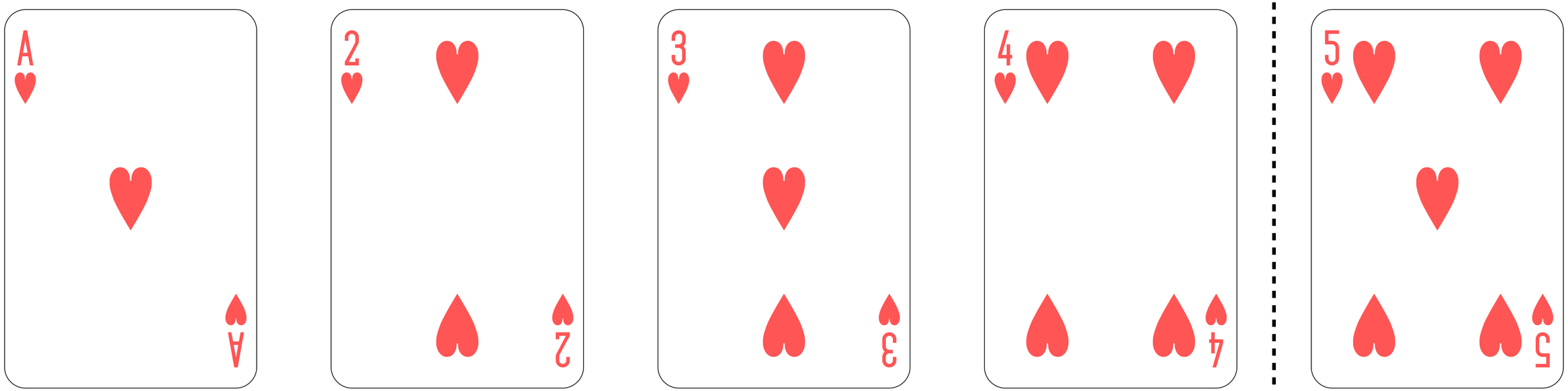

Sorting

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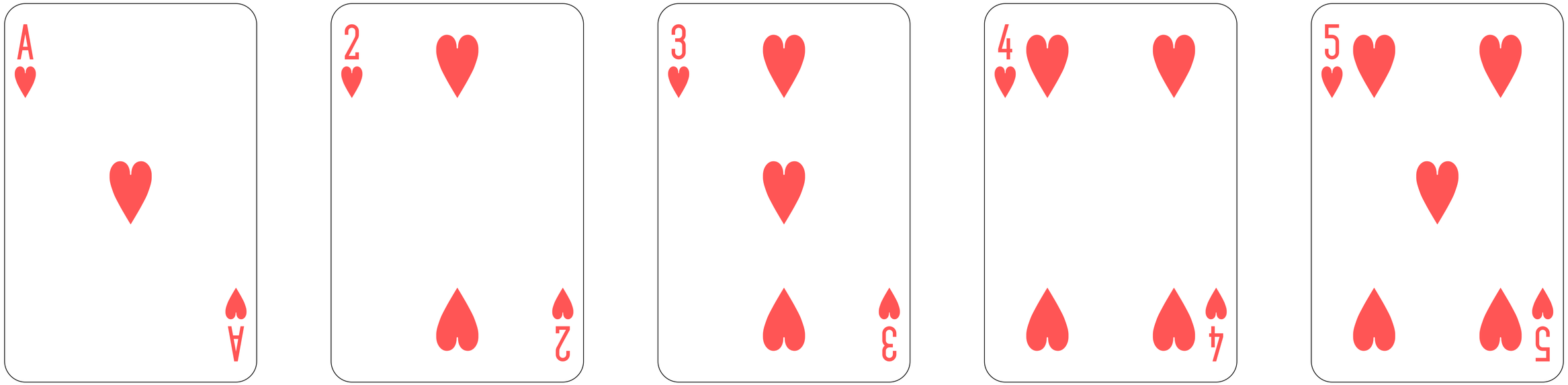
- Pick the smallest one and move it to the front



Sorting

Example 1 (how I do it): Algorithm

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For i = 1 to n - 1:
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Sorting

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- Why swap instead of pushing things over?

Sorting

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  - It's more efficient and we don't care about the order of the unsorted part

Sorting

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- Why swap instead of pushing things over?
  - It's more efficient and we don't care about the order of the unsorted part
- This is called *selection sort* -- we *select* the one we want repeatedly.

Sorting

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Sorting

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Sorting

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  - $(n-1)+(n-2)+\ldots+1 = n(n-1)/2 = O(n^2)$
- How many swaps?
  - $n-1$

Sorting

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- Everything left of the line is sorted.

Sorting

Example 1 (how I do it): Algorithm

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For i = 1 to n - 1:
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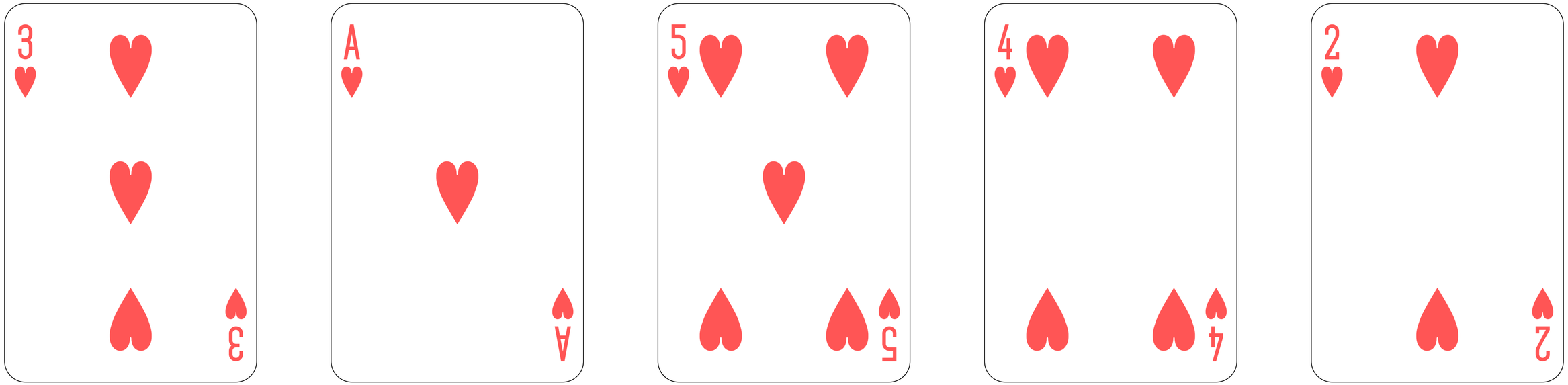
- Everything left of the line is sorted.
- Scanning the right (unsorted) part, and putting it to the end of the left (sorted) part.

Sorting

Example 2

Sorting

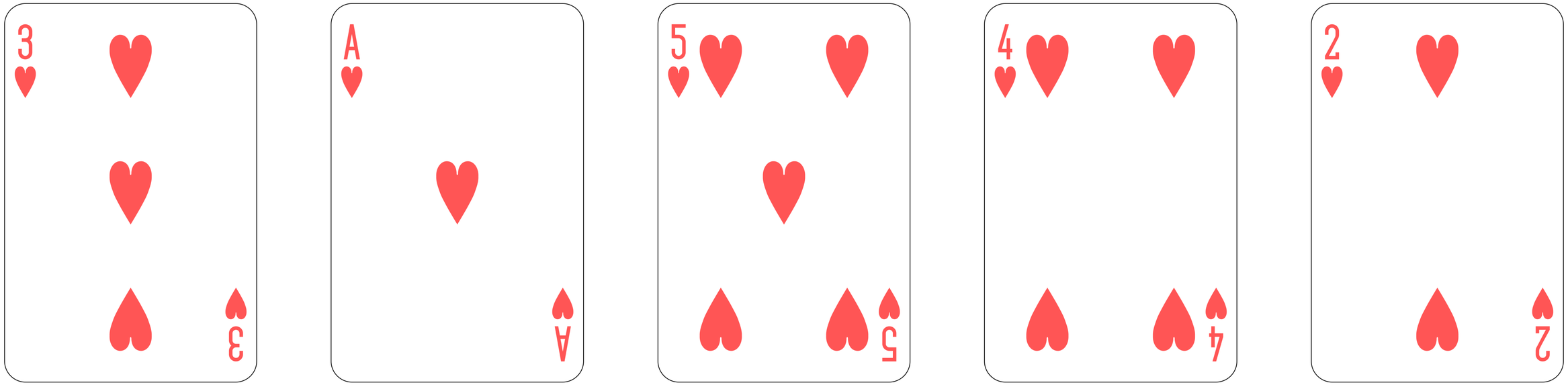
Example 2



Sorting

Example 2

- Pick the first unsorted and insert it into the right place



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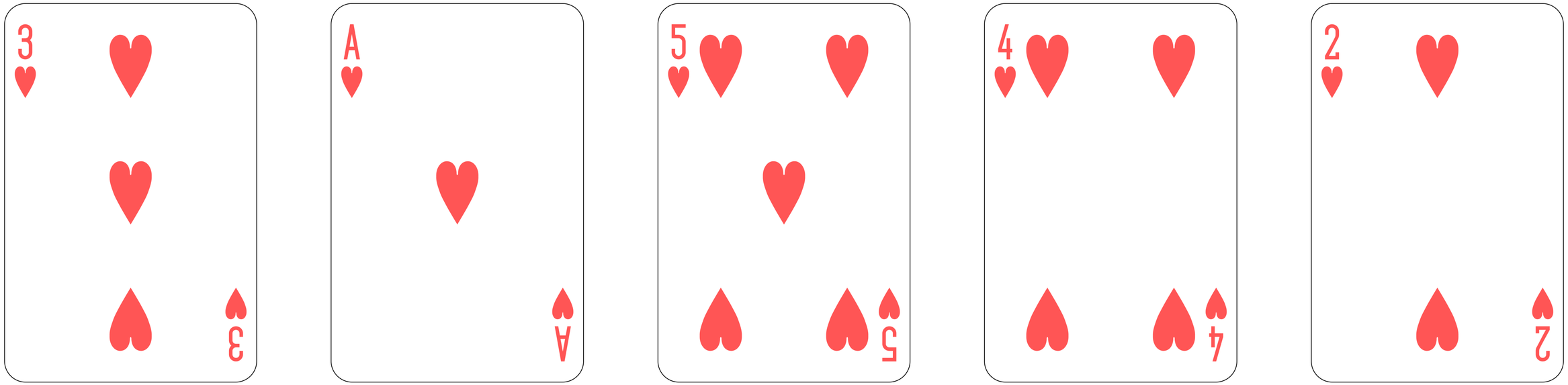

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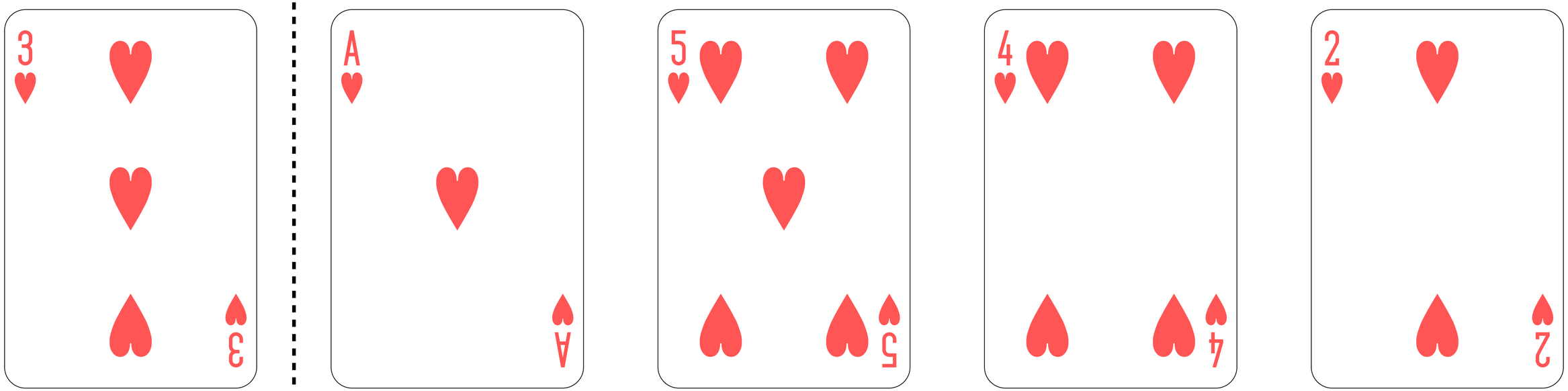
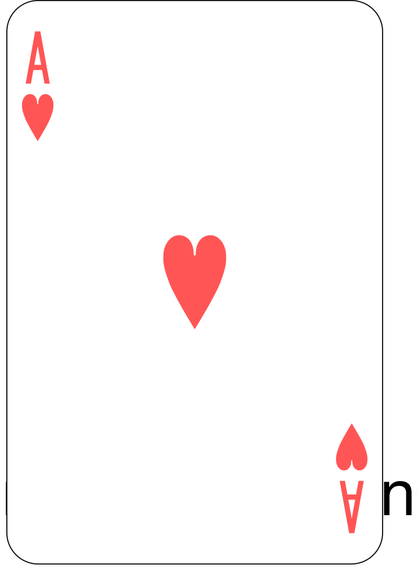


Sorting

Example 2



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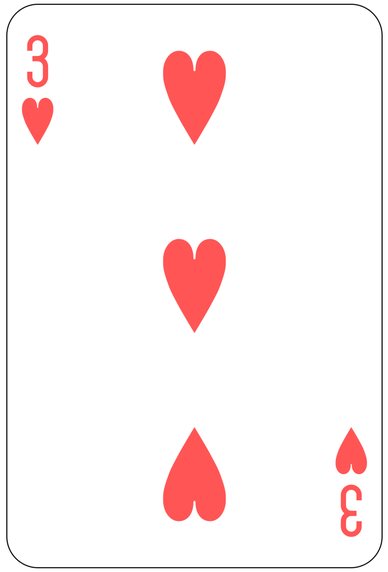

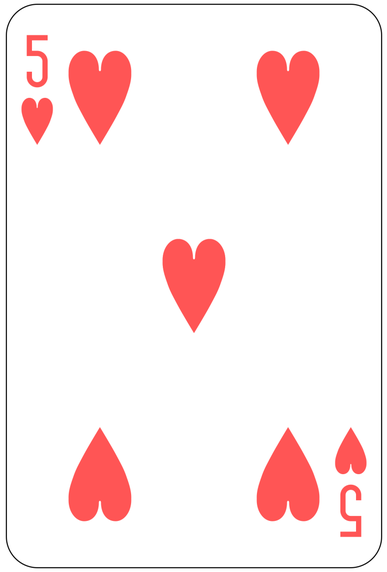

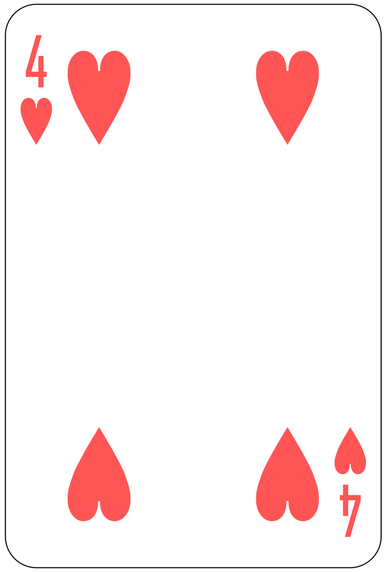

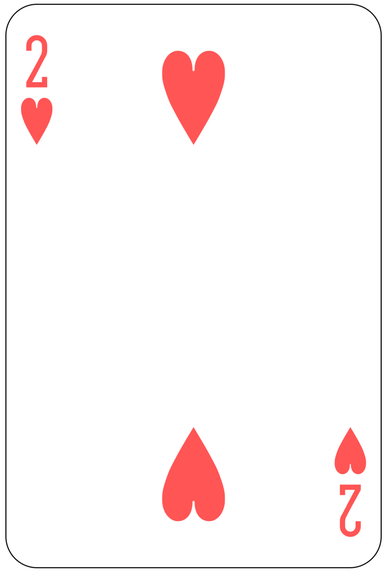

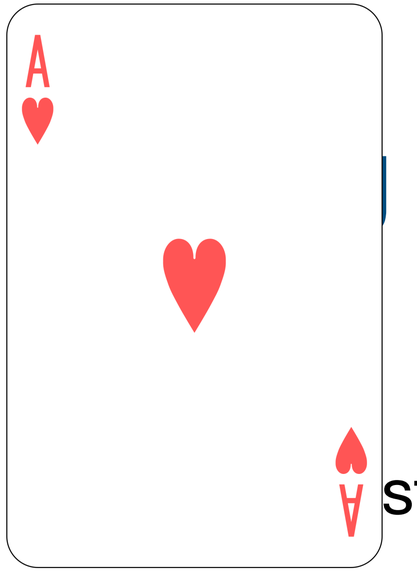

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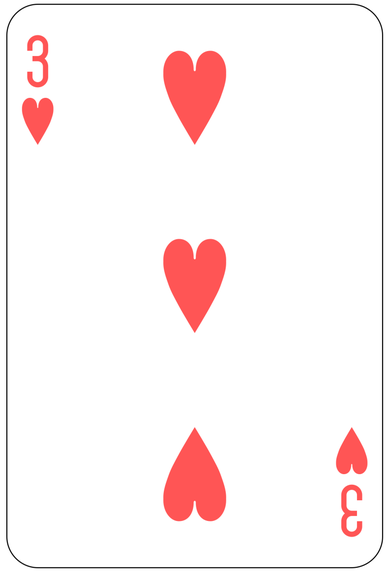

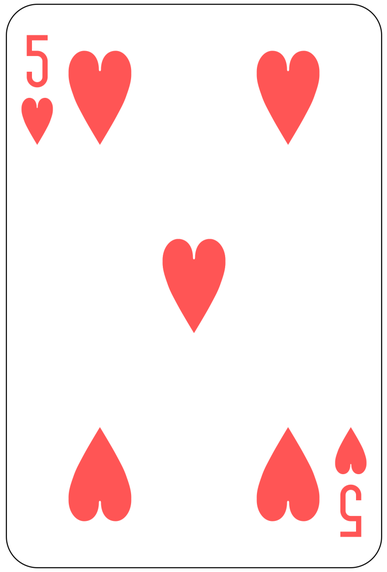

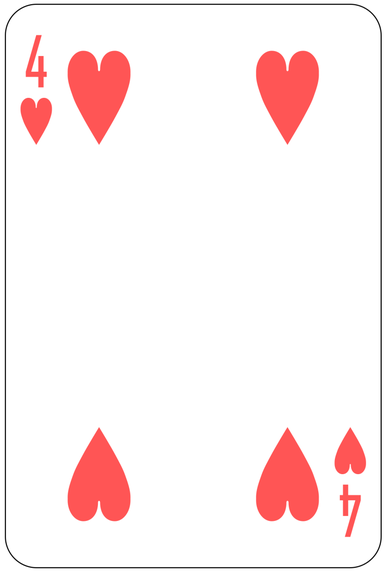

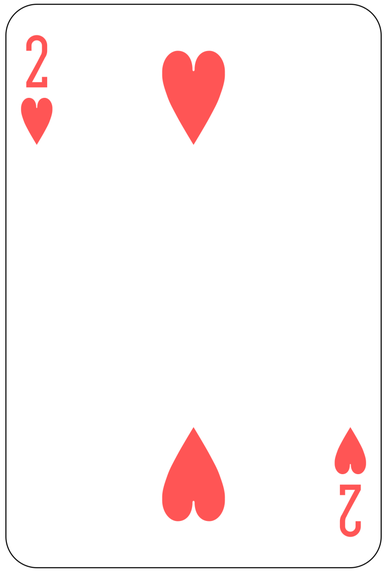

Sorting

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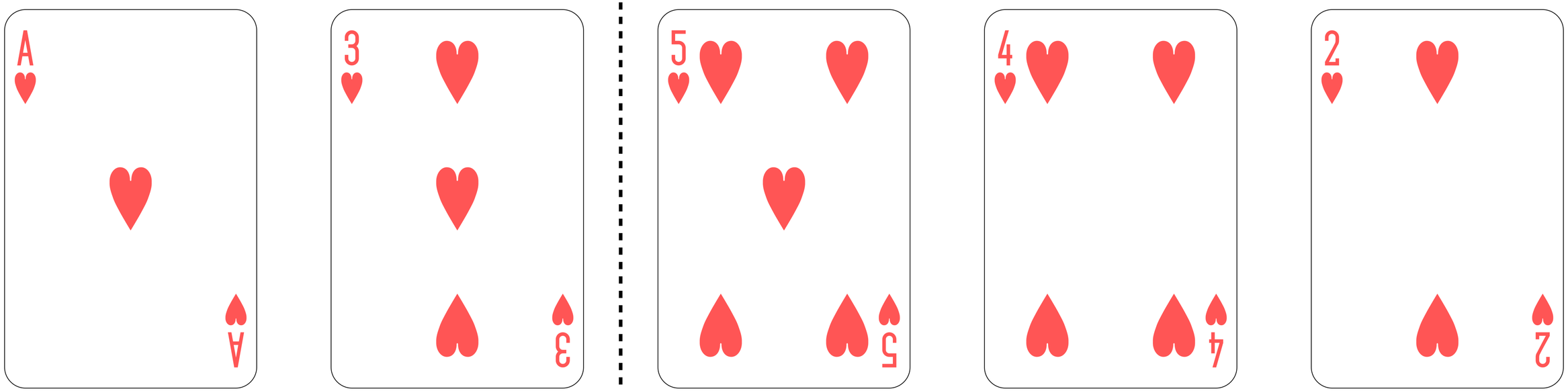

Sorting

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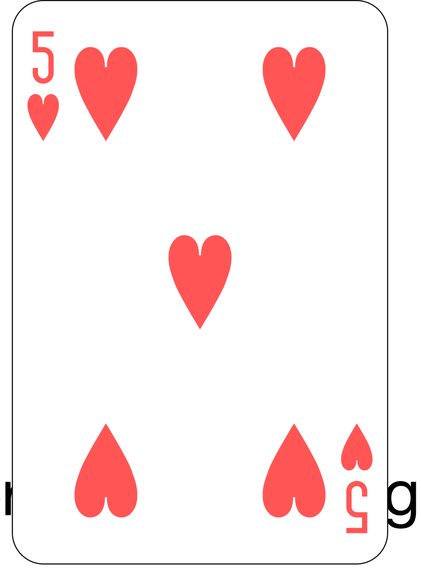
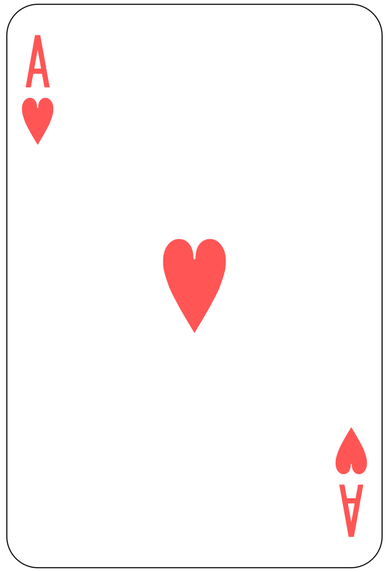
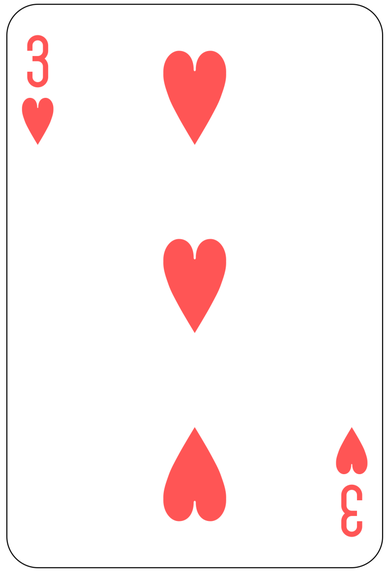
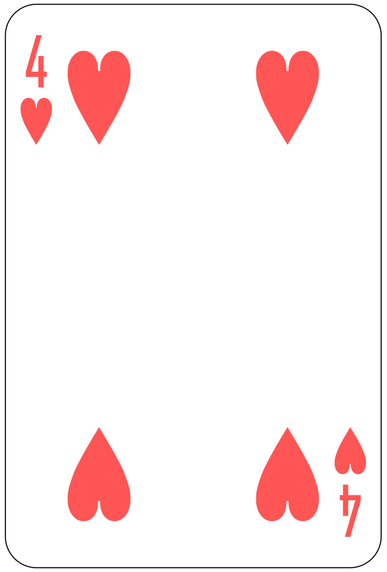
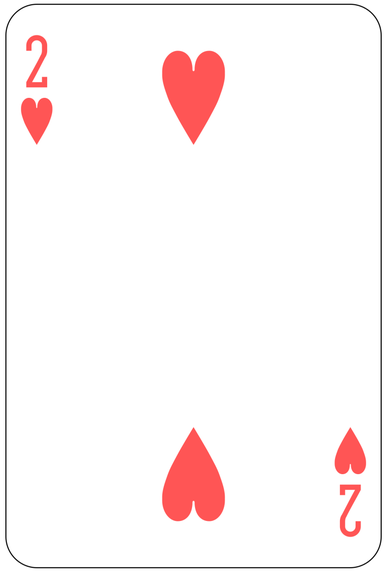




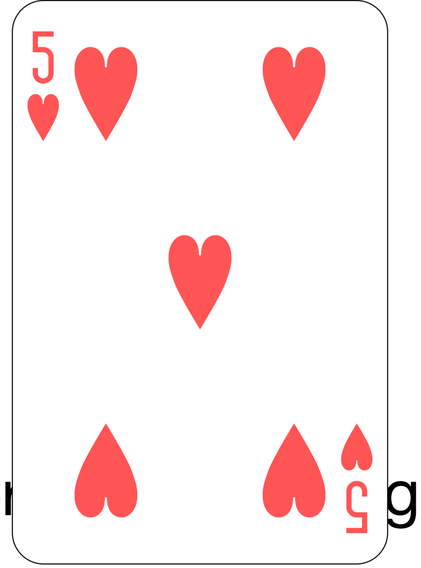






Sorting

Example 2



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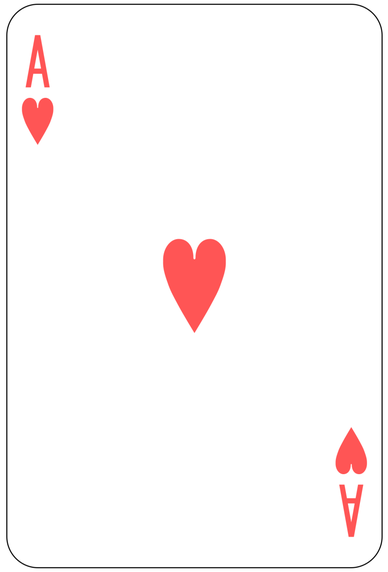

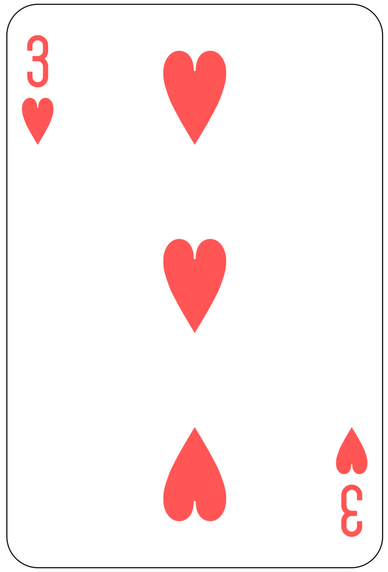

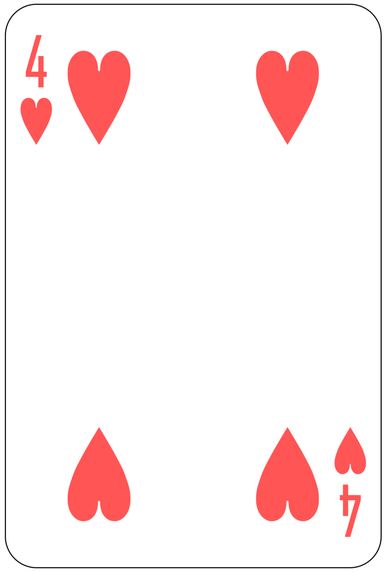

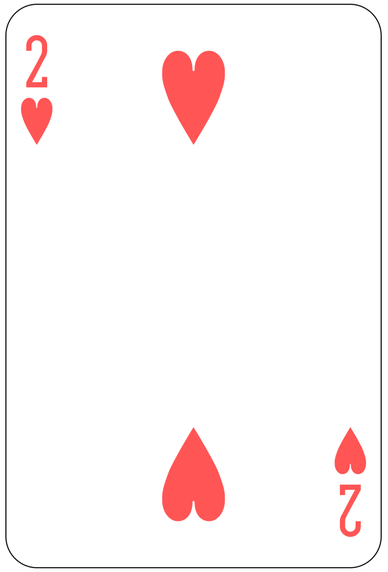

Sorting

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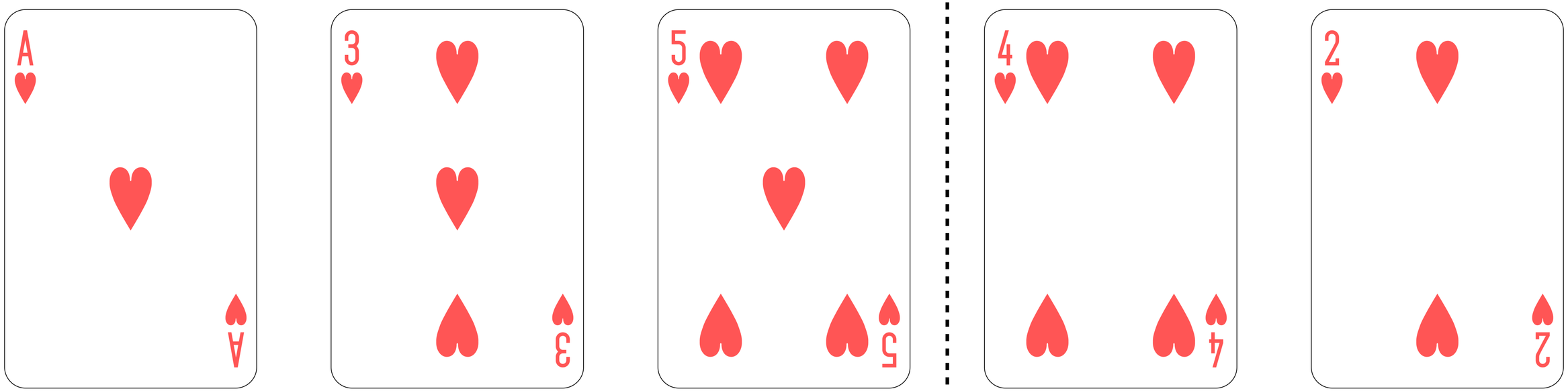

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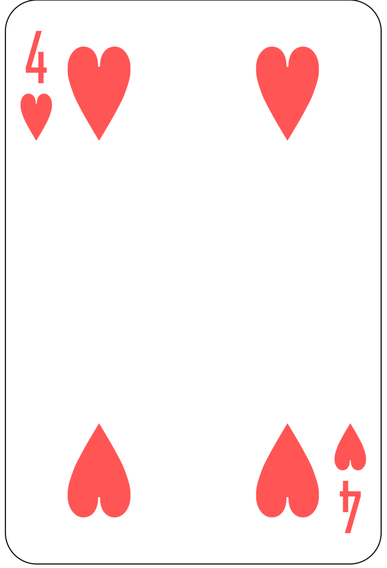

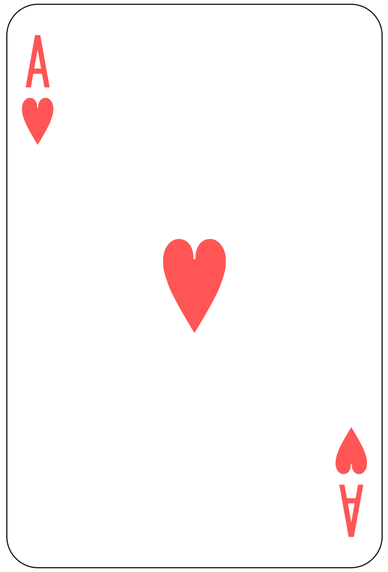

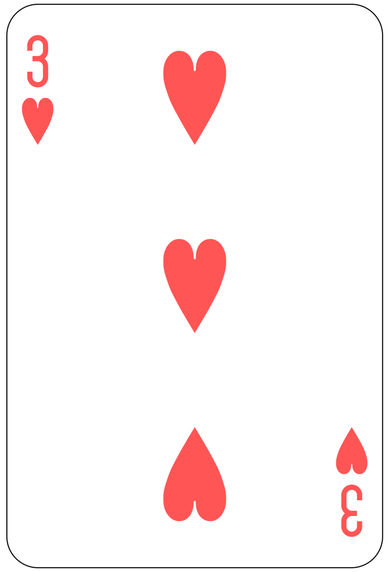

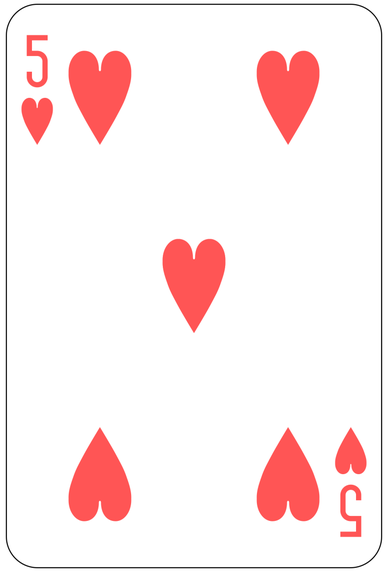

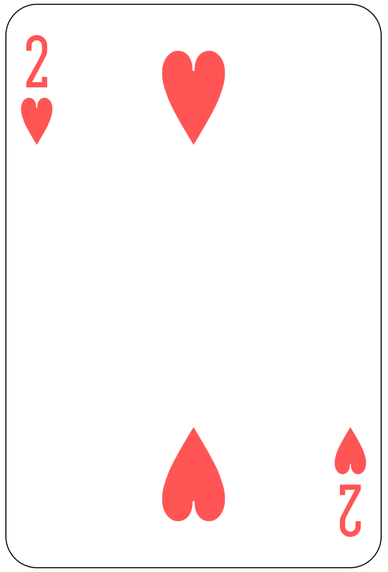

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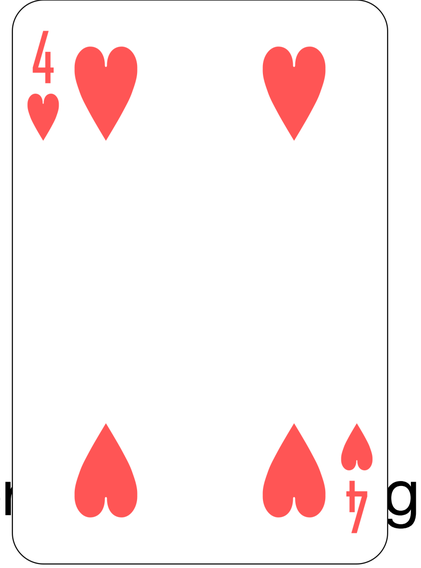

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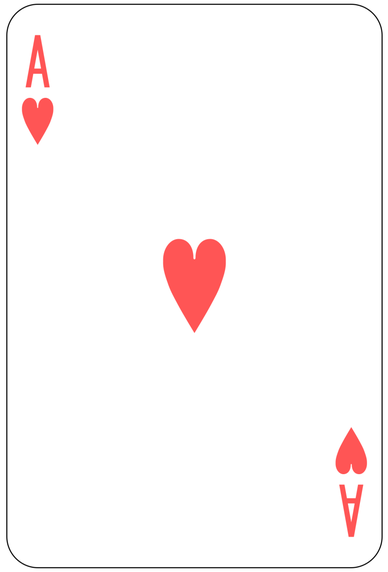

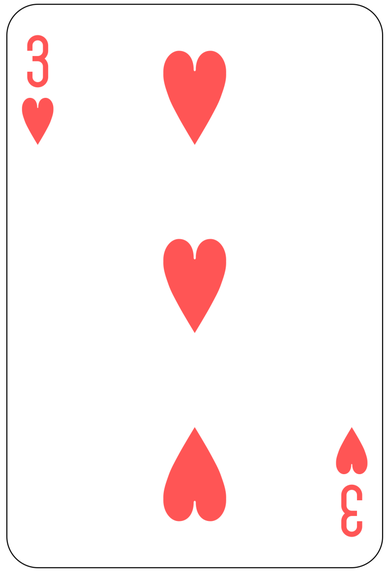

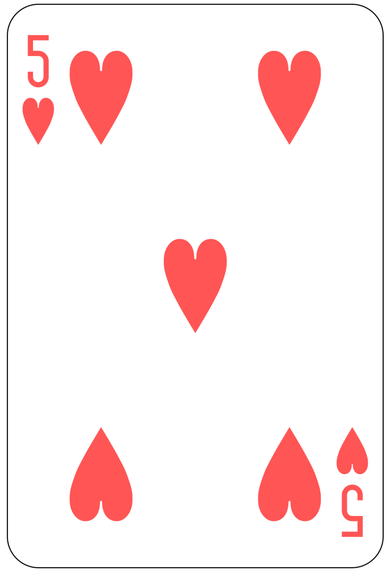

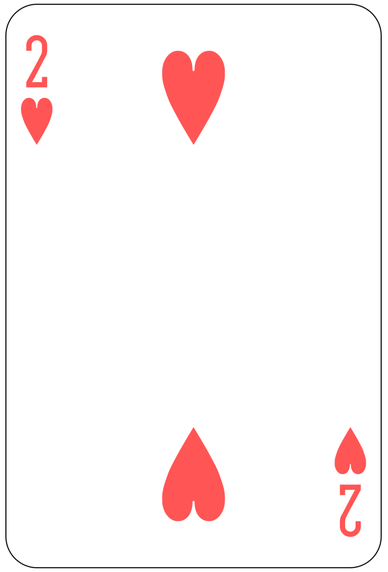

Sorting

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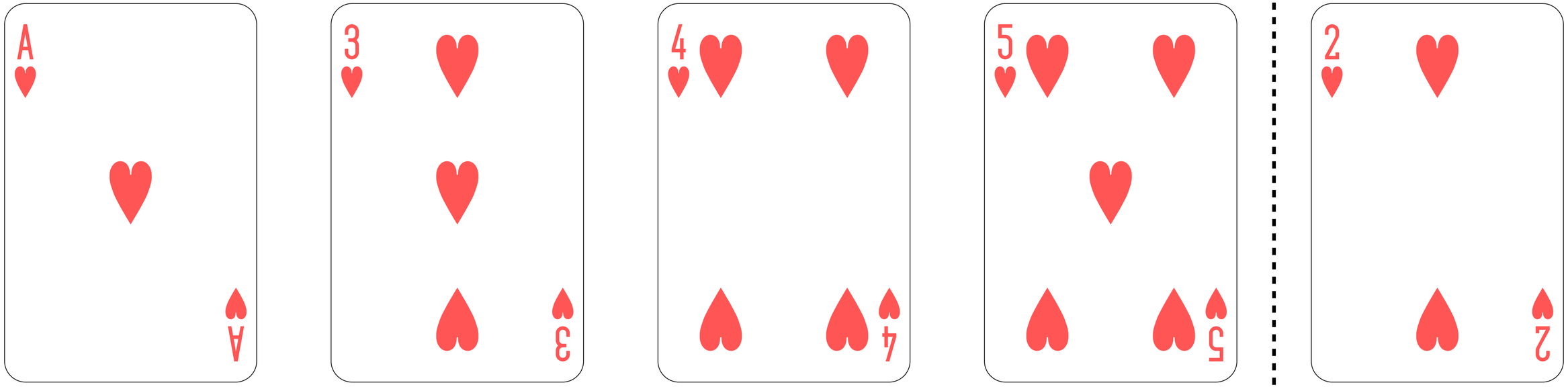

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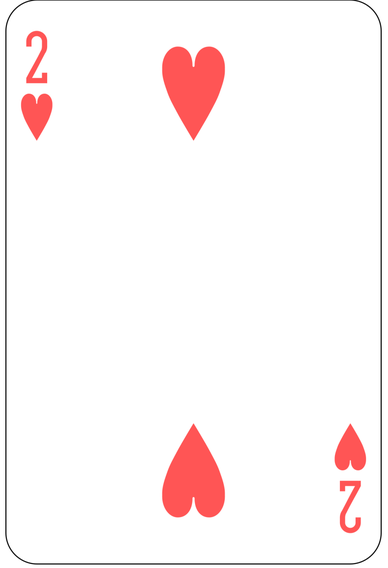

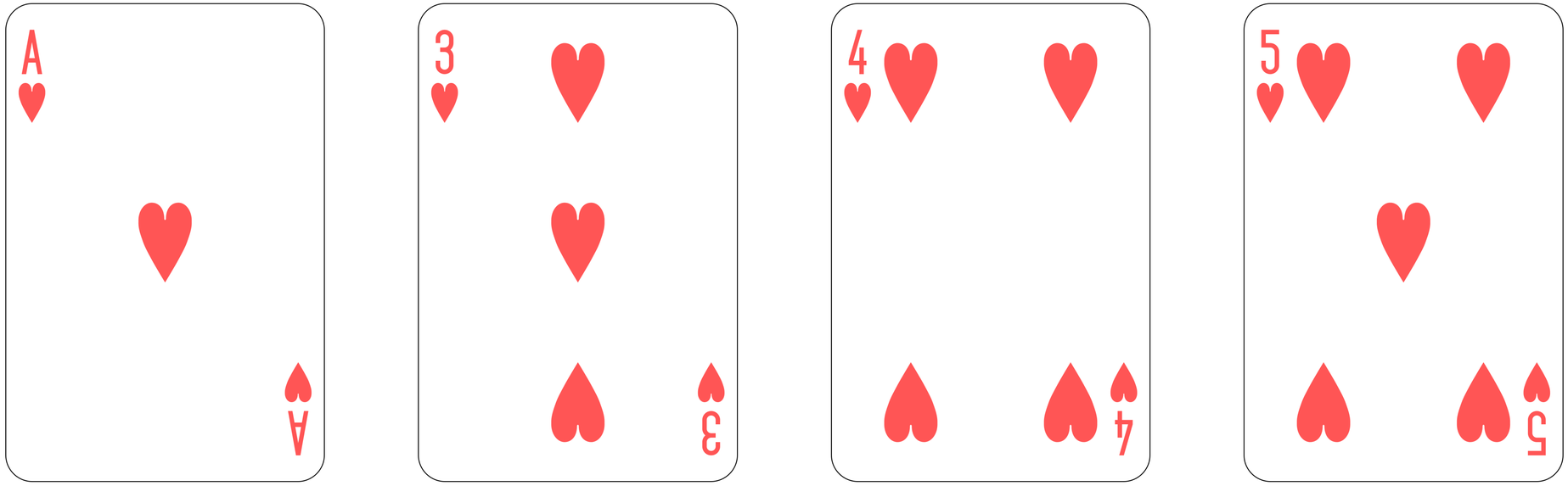

Sorting

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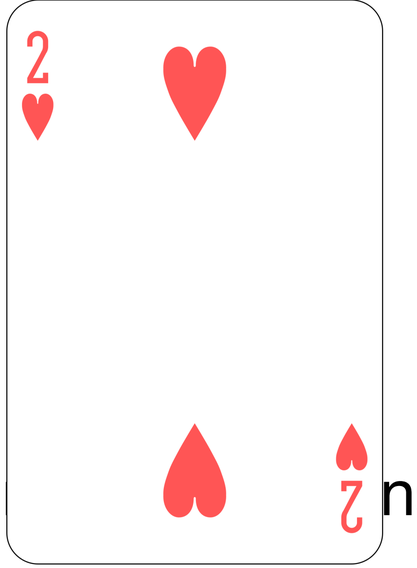

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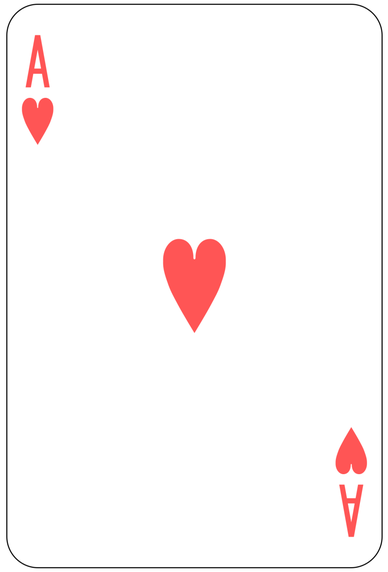
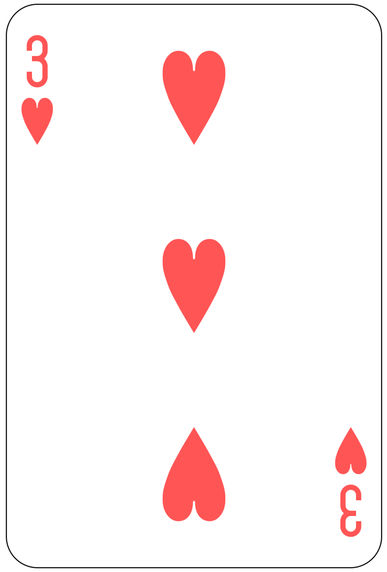
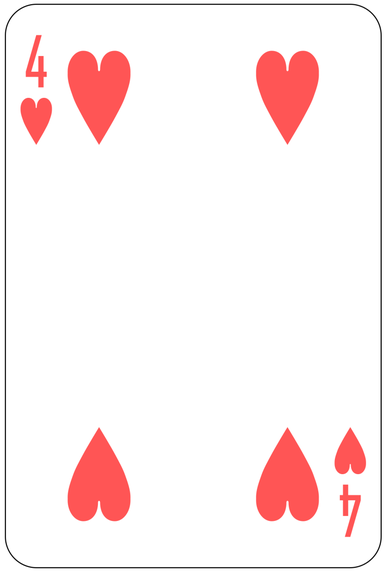
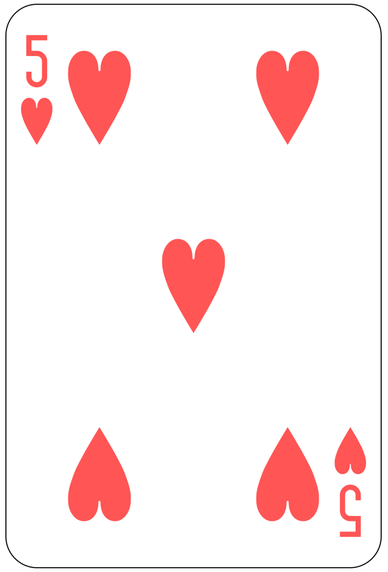




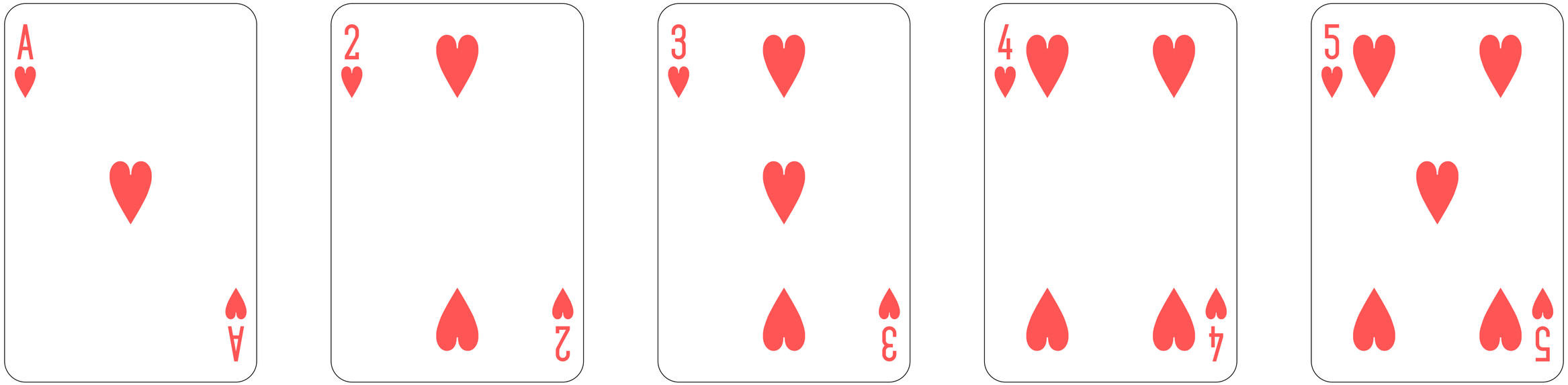




Sorting

Example 2

- Pick the first unsorted and insert it into the right place



Sorting

Example 2: Algorithm

```
for i = 2 to n:
    j = i - 1
    while j > 0 and A[j] > A[i]:
        A[j + 1] = A[j]
        j = j - 1
    A[j] = A[i]
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Sorting

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- Take the first unsorted and insert it into the right place in the sorted pile

Sorting

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- Take the first unsorted and insert it into the right place in the sorted pile
- Inserting means shifting everything after the card by one place

Sorting

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- Take the first unsorted and insert it into the right place in the sorted pile
- Inserting means shifting everything after the card by one place
- This is called *insertion sort*.

Sorting

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- Comparison: worst-case $O(n^2)$

Sorting

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- Comparison: worst-case $O(n^2)$
- Swap: worst-case $O(n^2)$

Sorting

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        j = j - 1
    A[j] = A[i]
```

- Everything left of the line is sorted

Sorting

Example 2: Algorithm

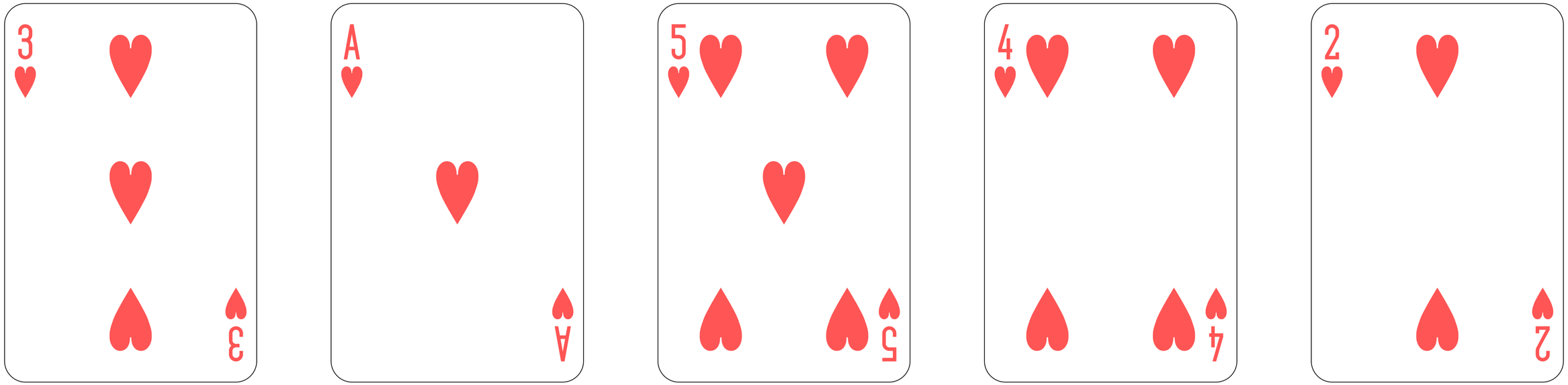
```
For i = 2 to n:
    j = i - 1
    while j > 0 and A[j] > A[i]:
        A[j + 1] = A[j]
        j = j - 1
    A[j] = A[i]
```

- Everything left of the line is sorted
- Take the first one on the right, scanning the left to find a place.

Sorting

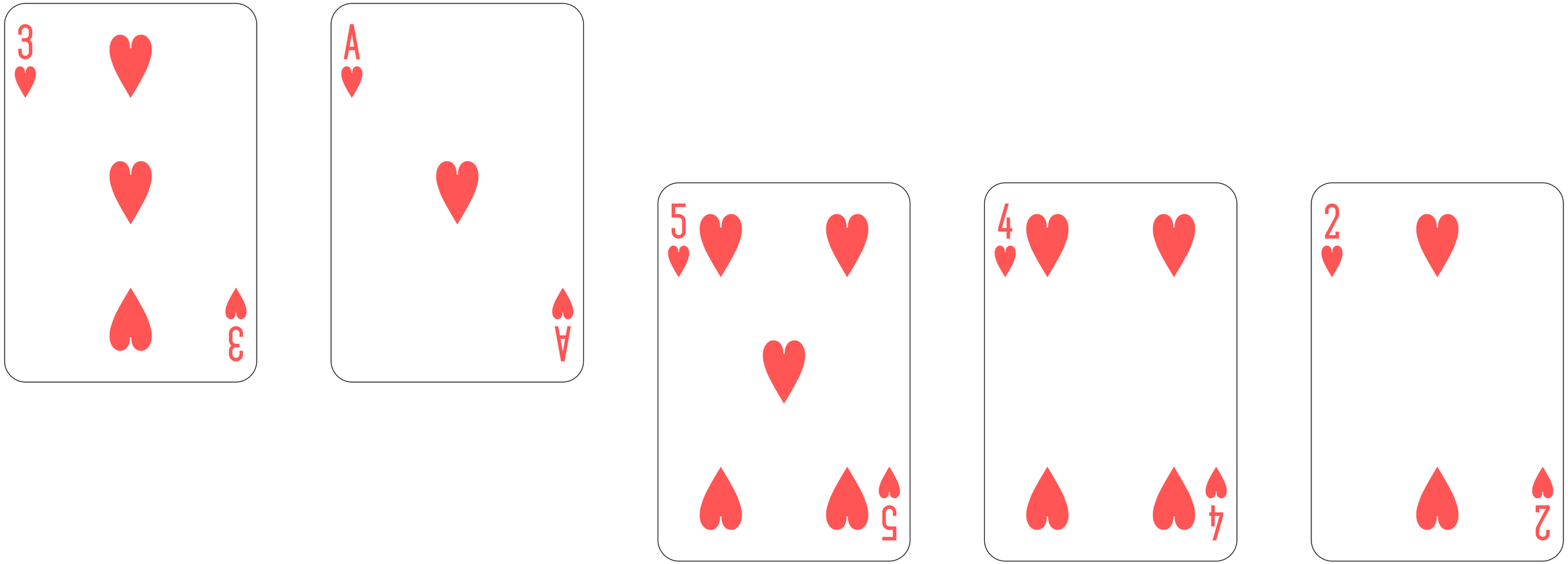
Example 3

- Take two cards, swap them if out of order.



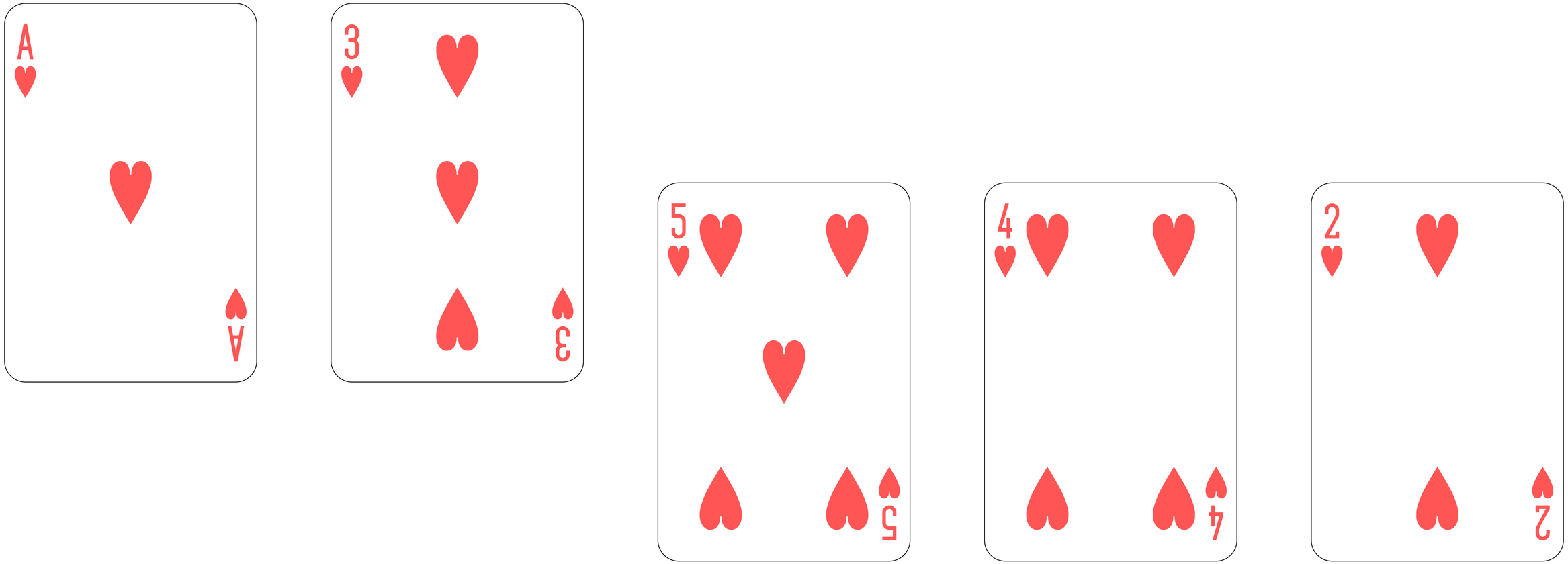
Sorting

Example 3



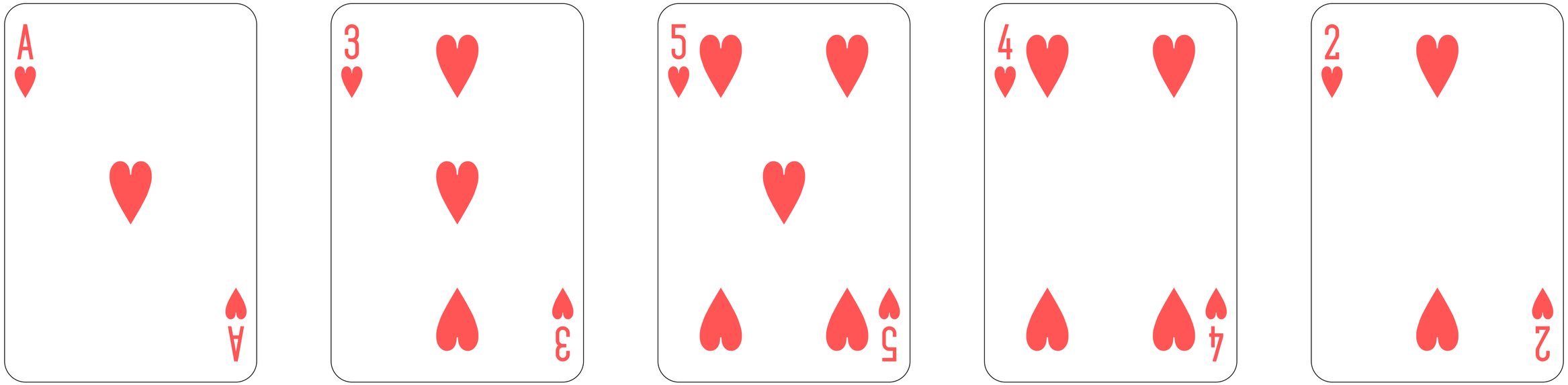
Sorting

Example 3



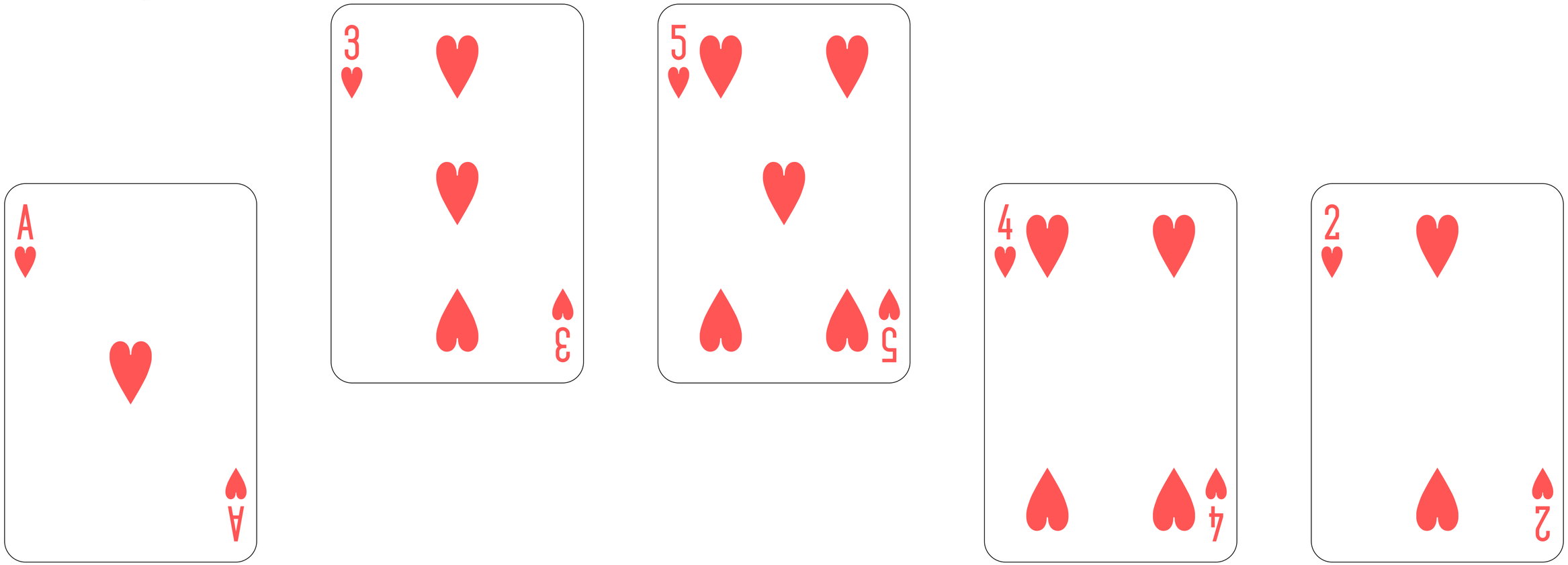
Sorting

Example 3



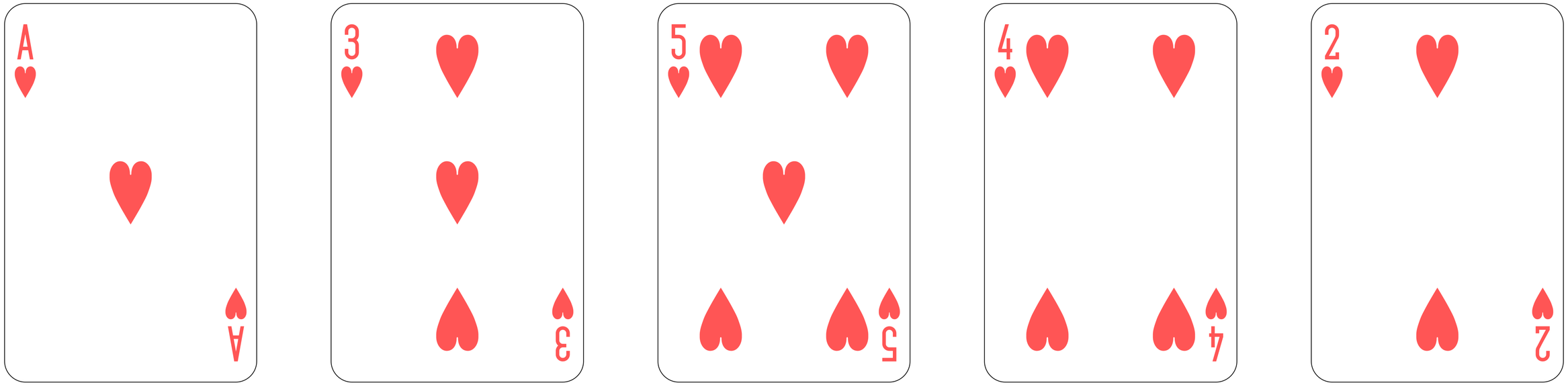
Sorting

Example 3



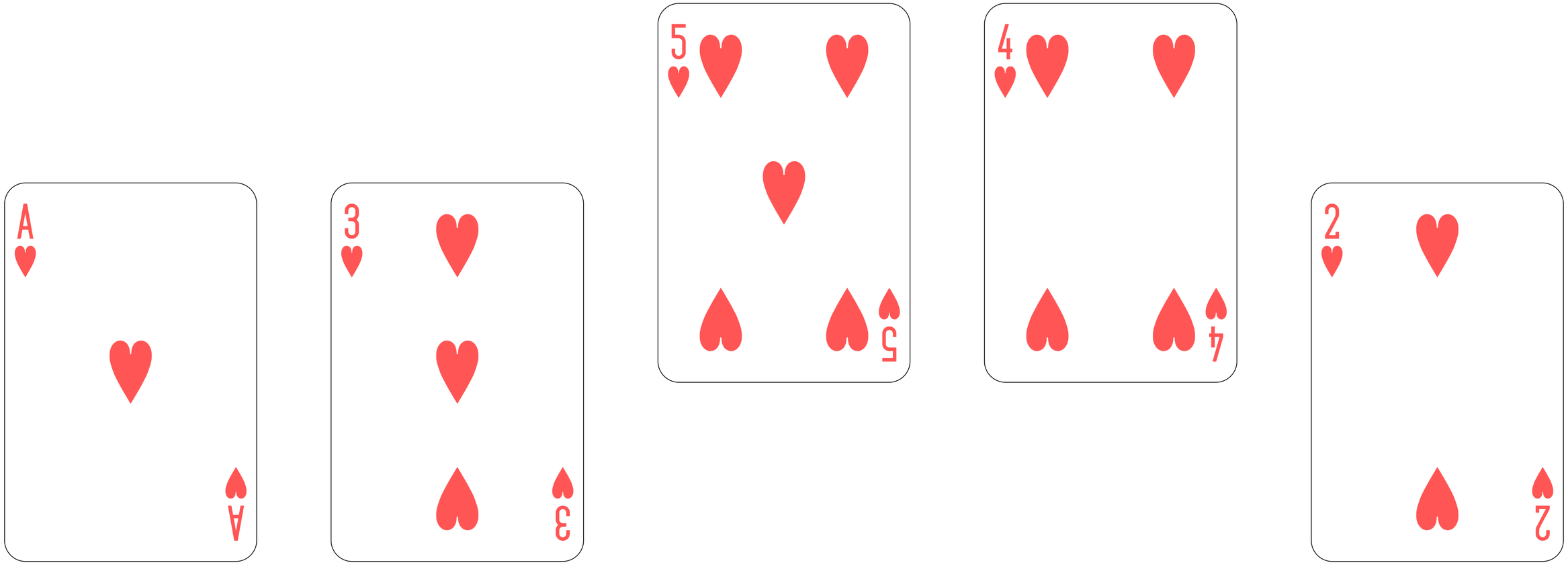
Sorting

Example 3



Sorting

Example 3



Sorting

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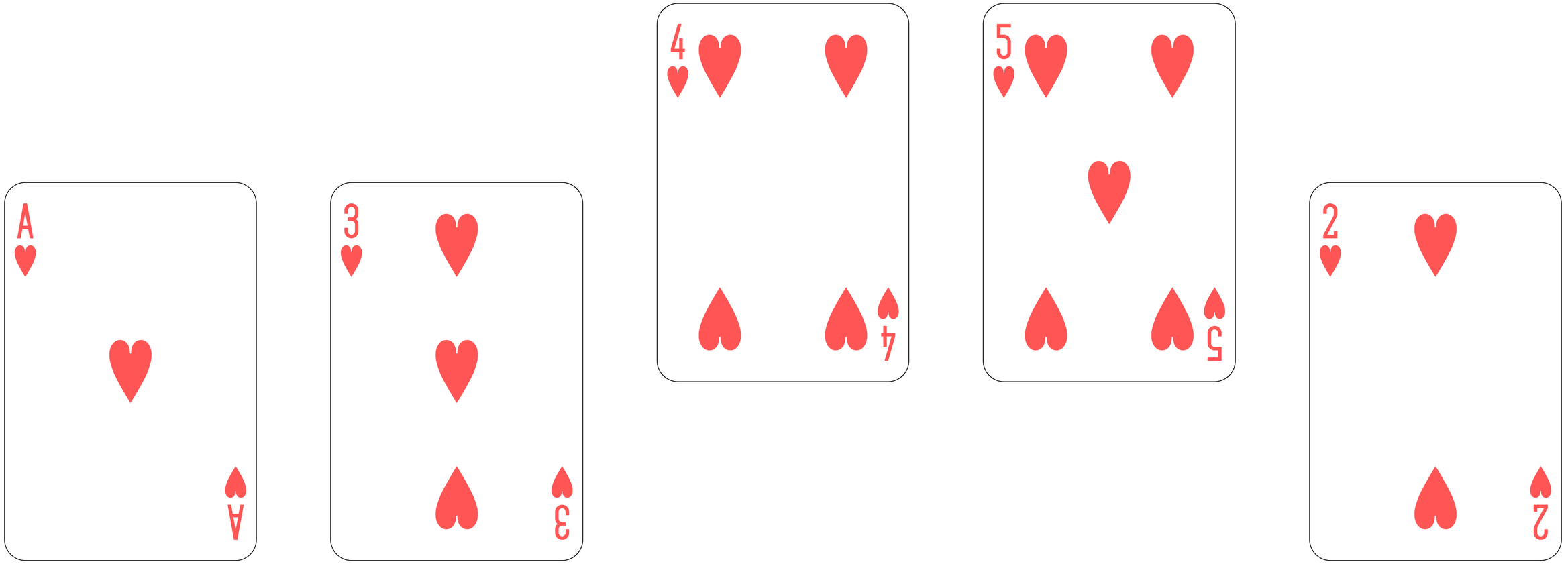
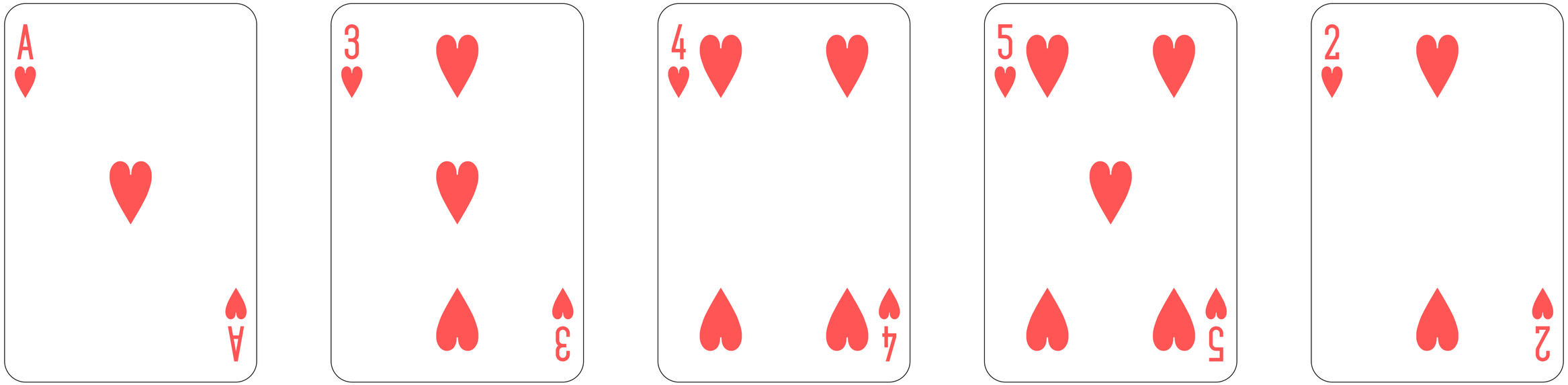


Sorting

Example 3



Sorting

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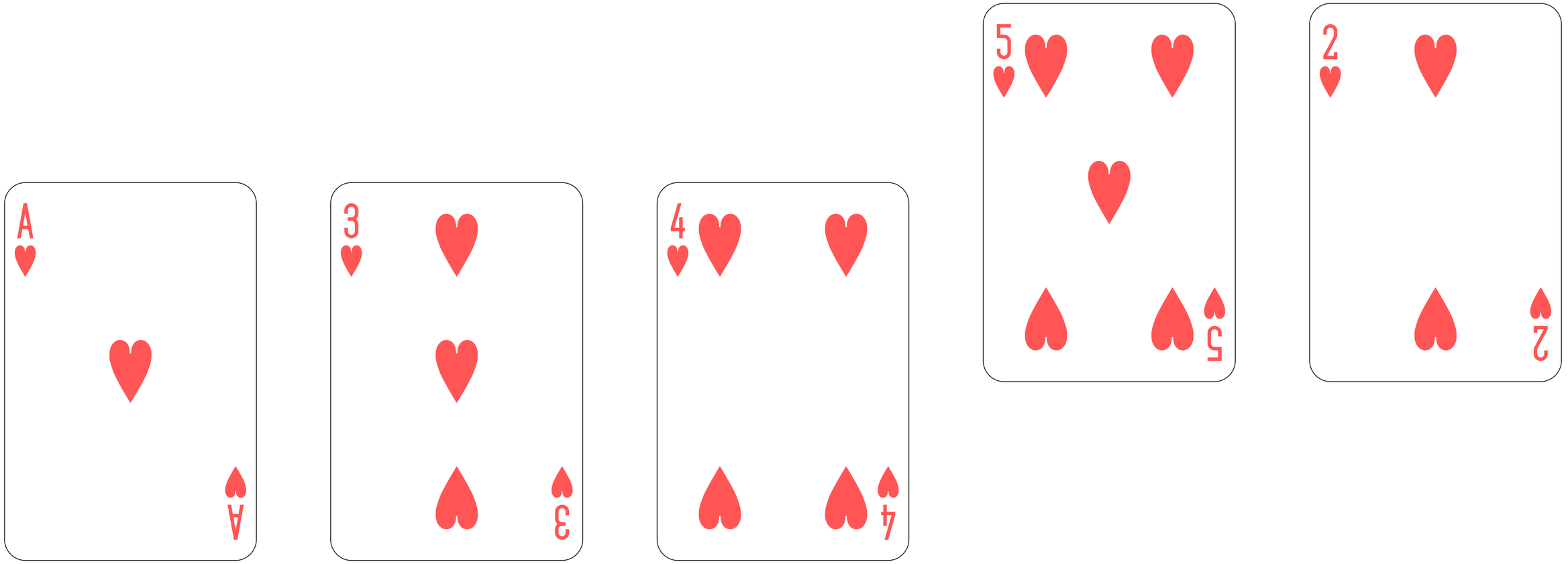

Sorting

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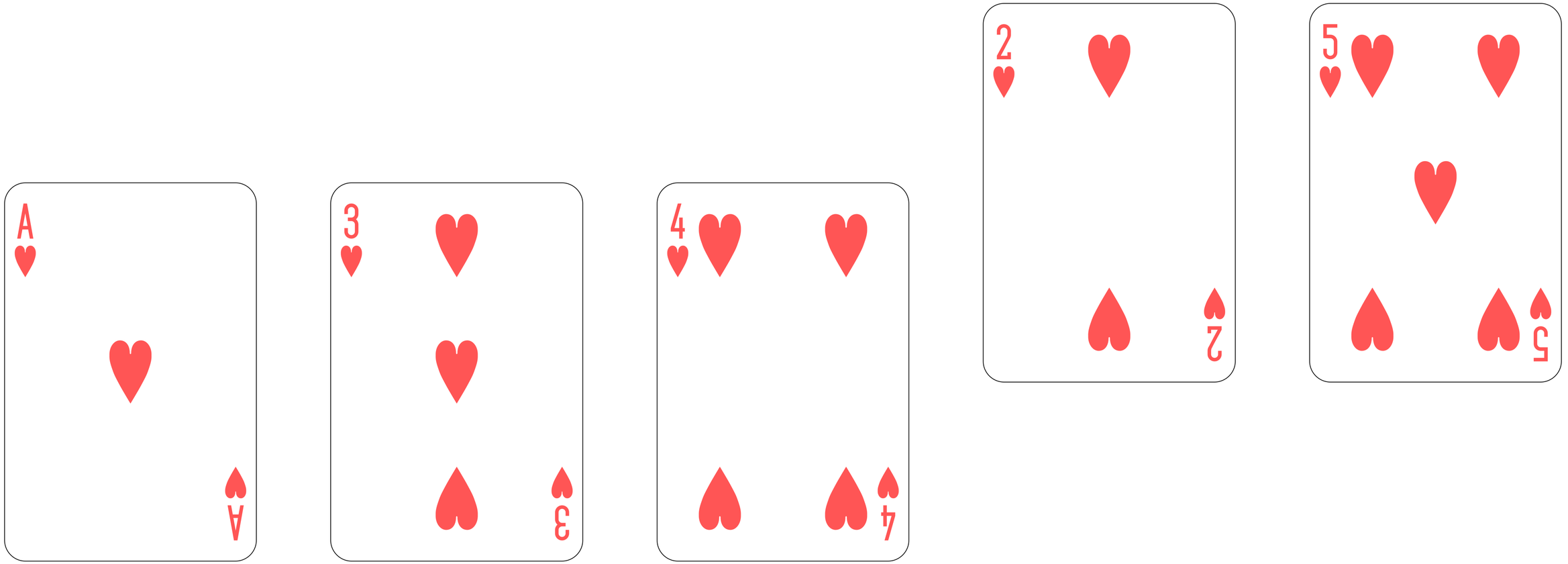

Sorting

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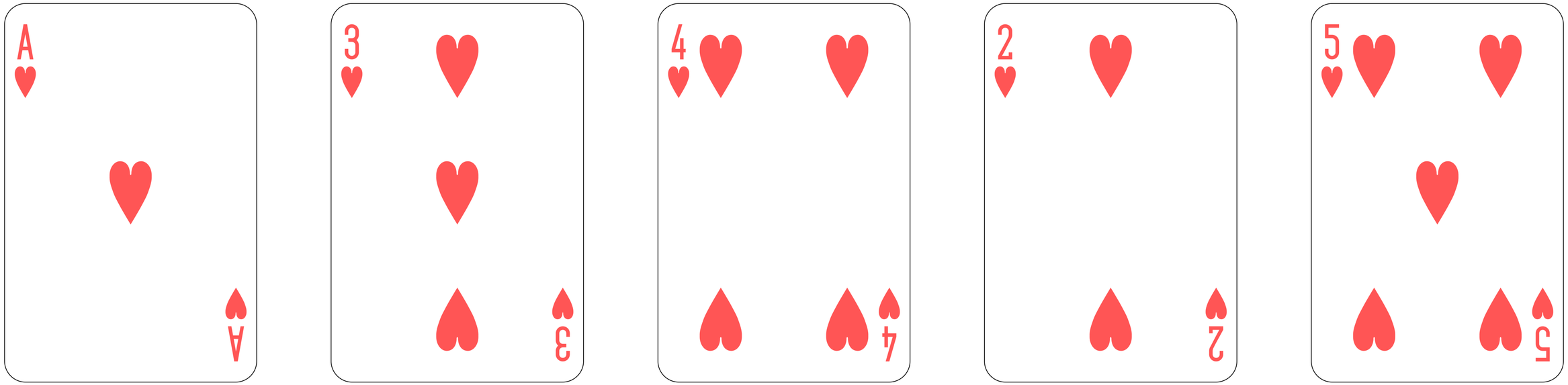
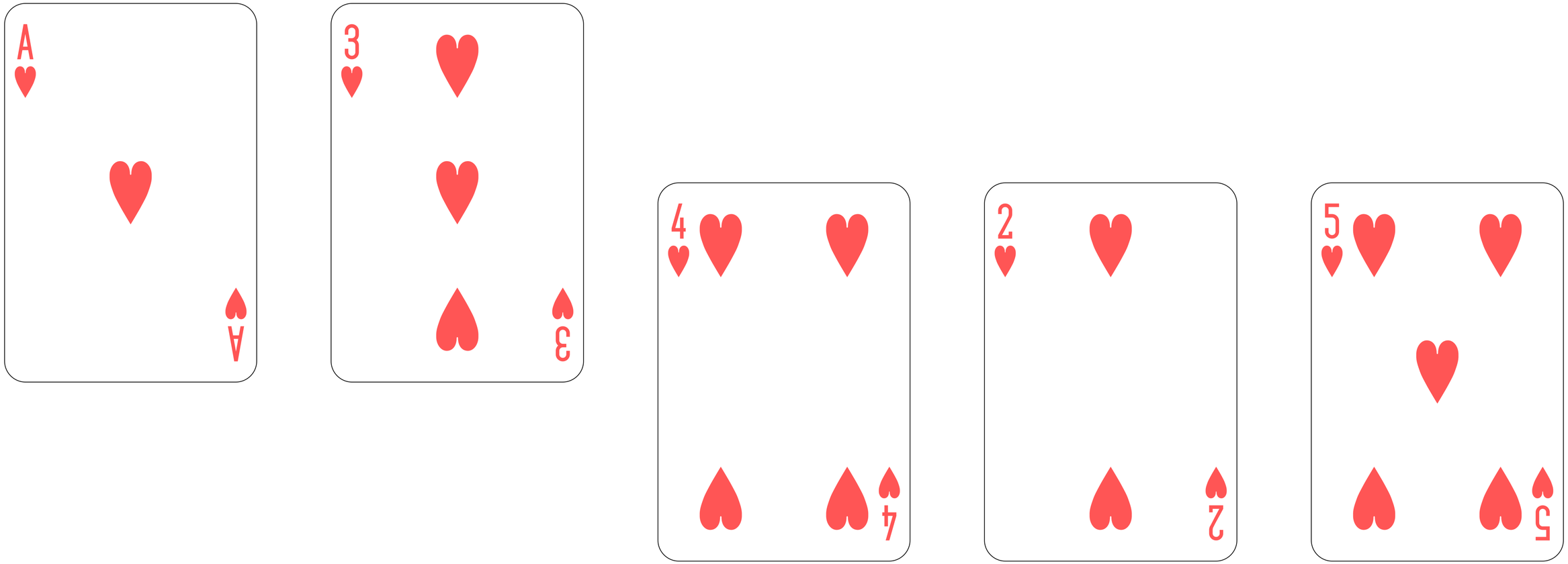


Sorting

Example 3



Sorting

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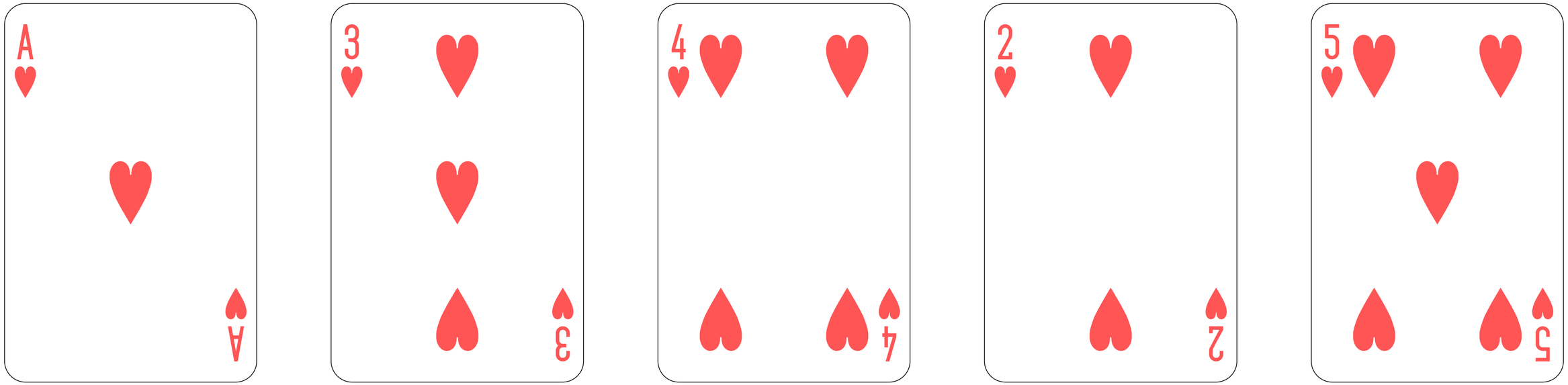

Sorting

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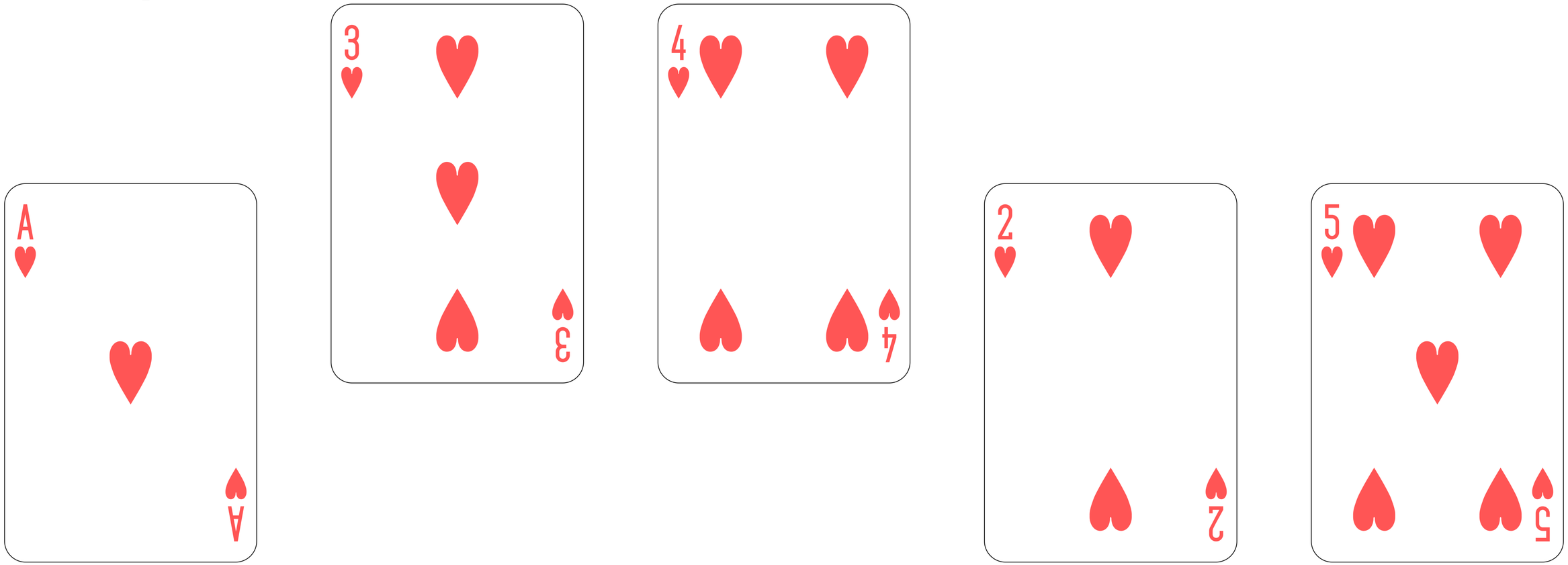
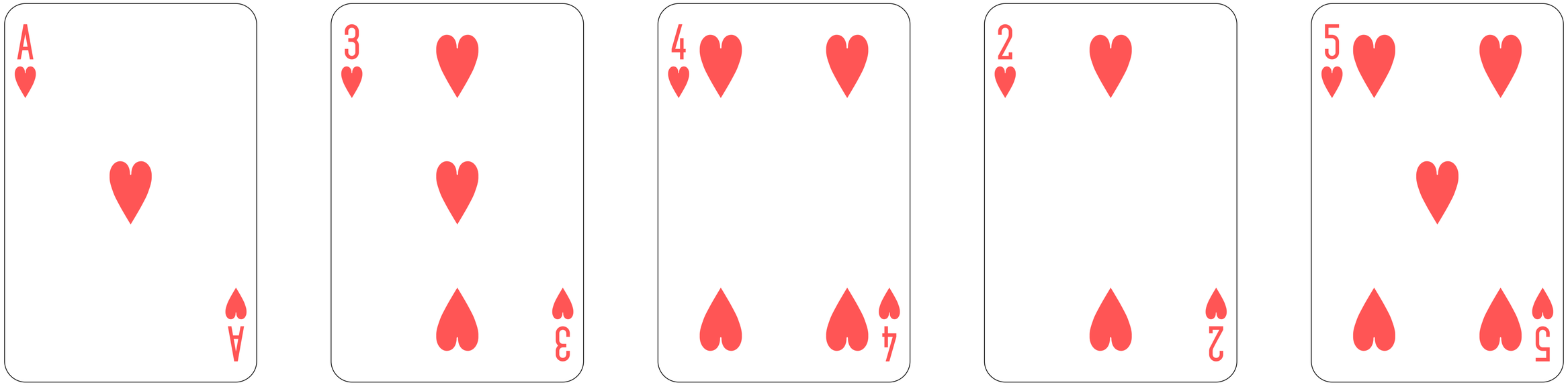


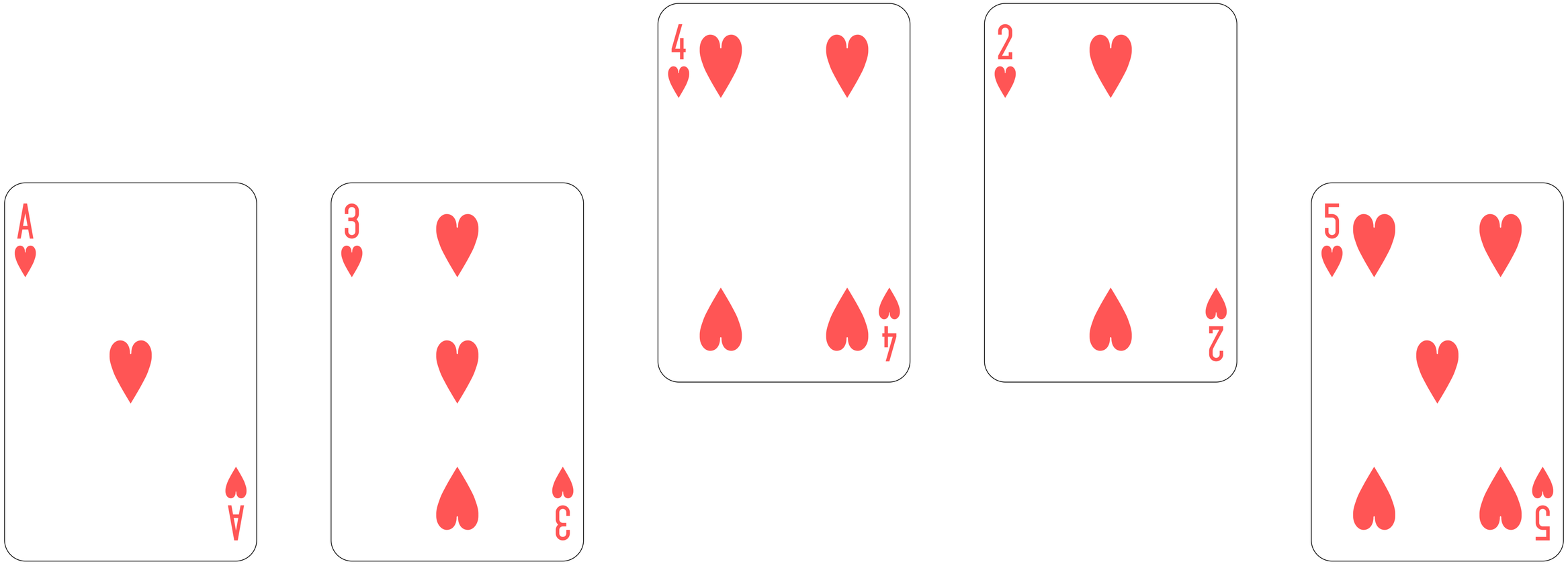
Sorting

Example 3



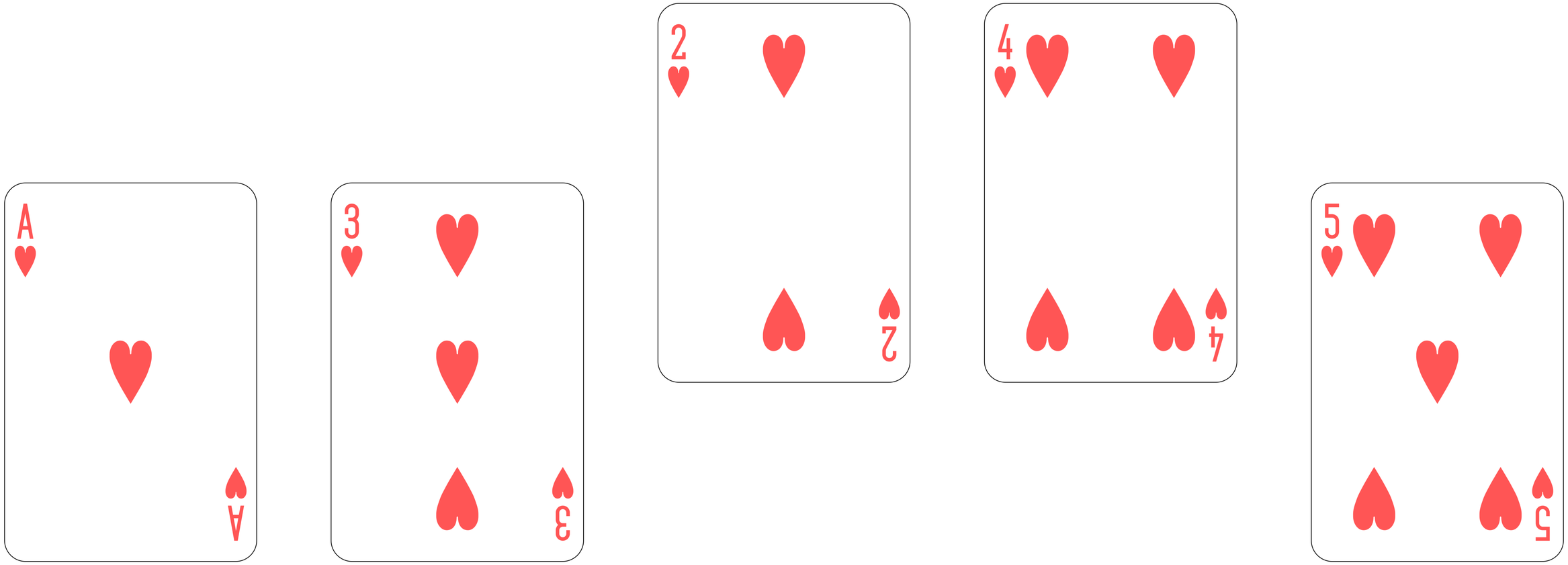
Sorting

Example 3



Sorting

Example 3



Sorting

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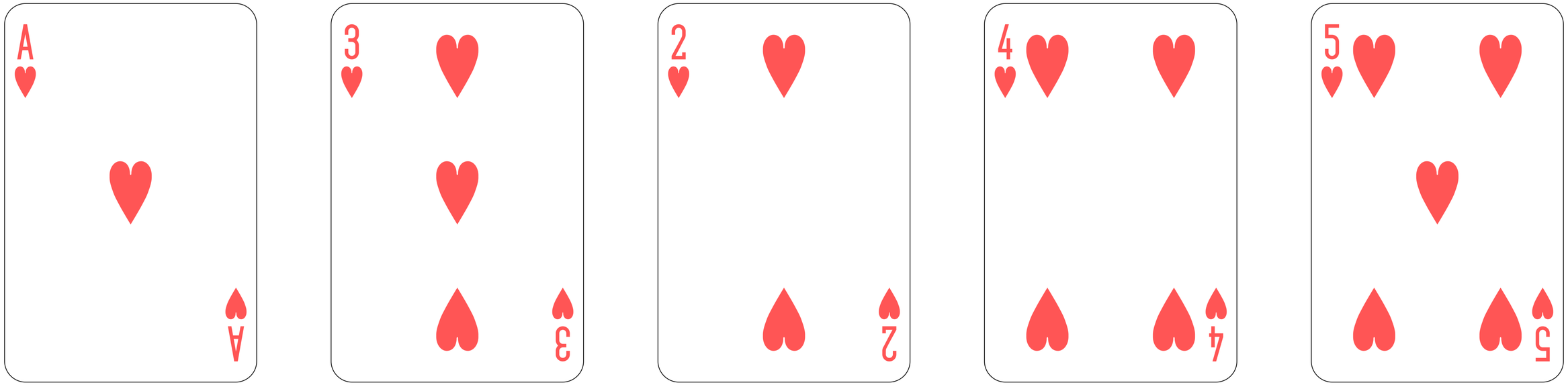

Sorting

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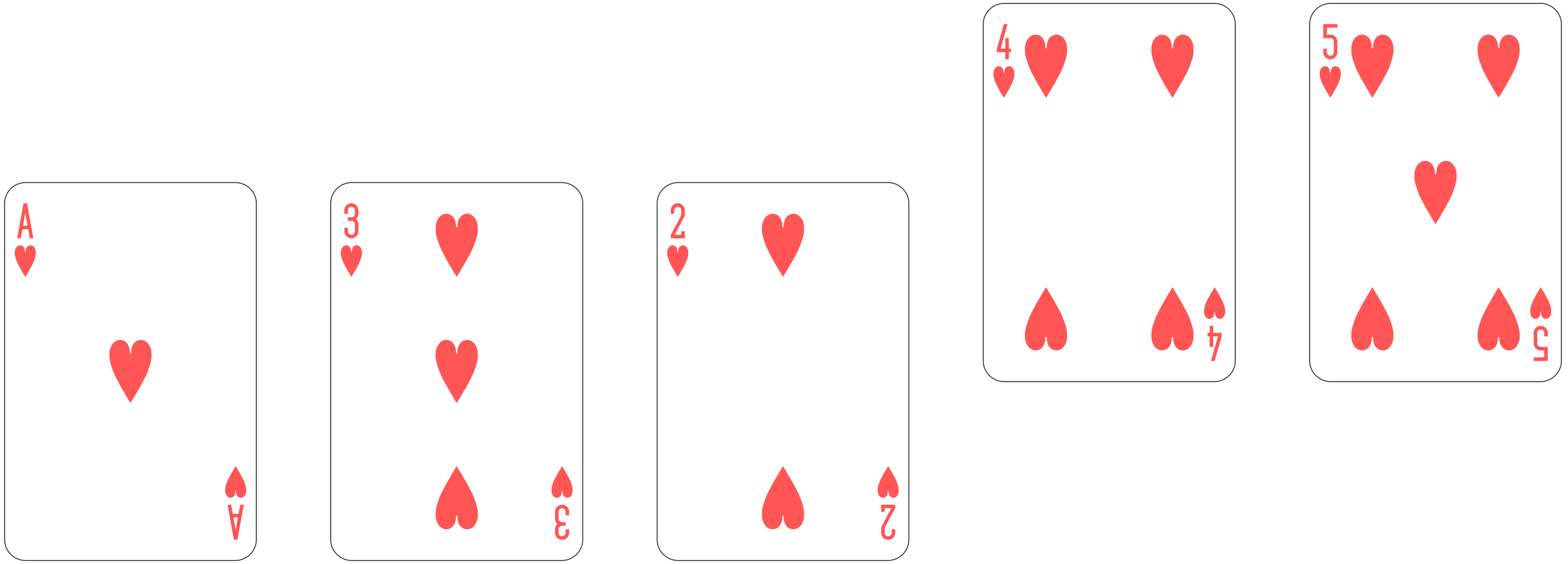

Sorting

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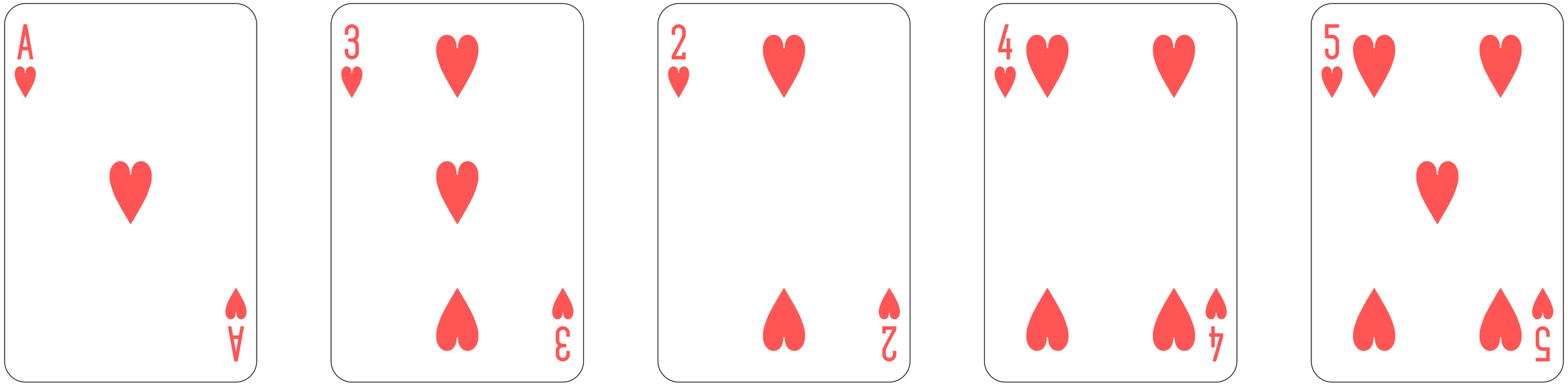

Sorting

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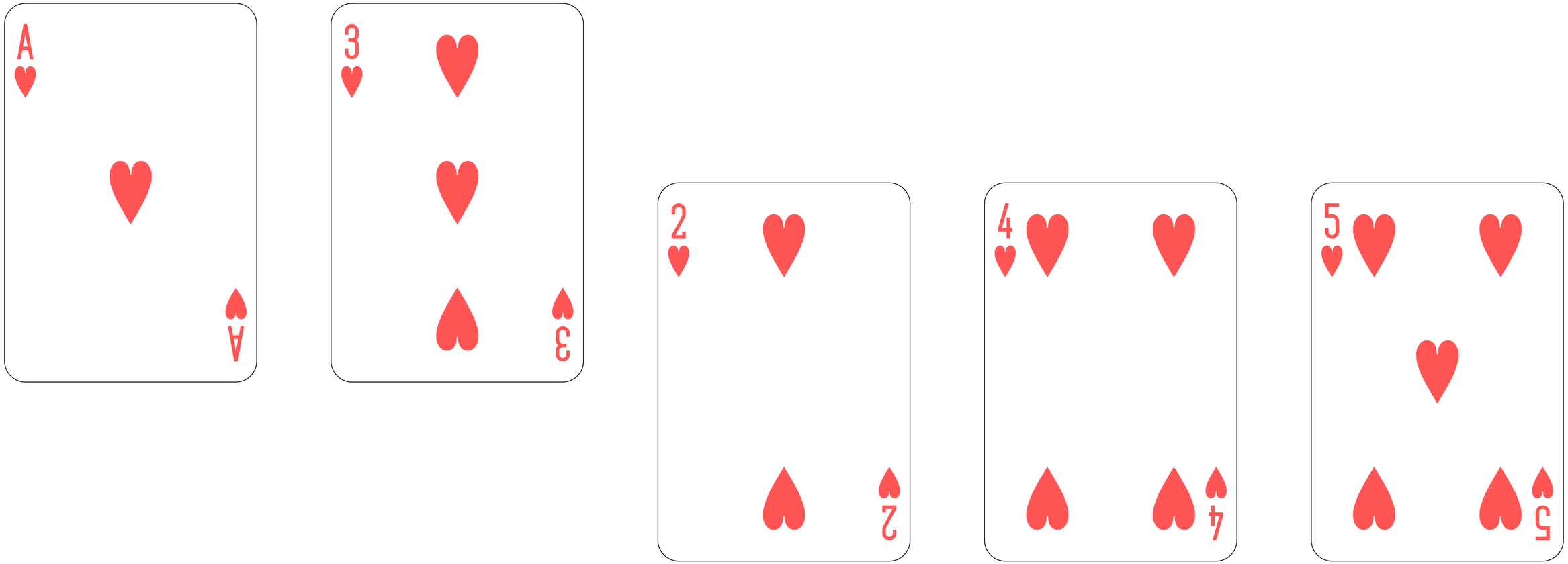

Sorting

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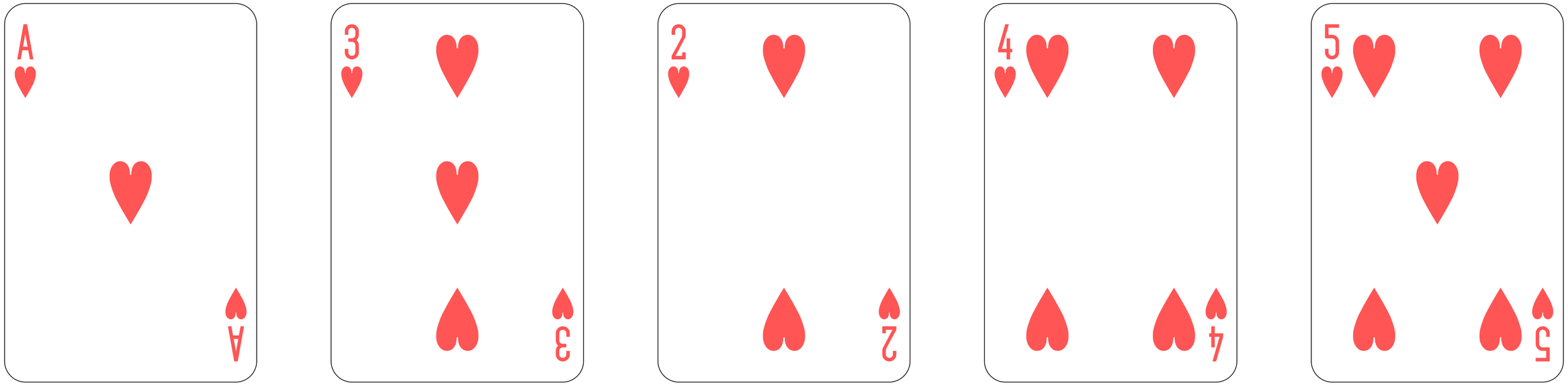

Sorting

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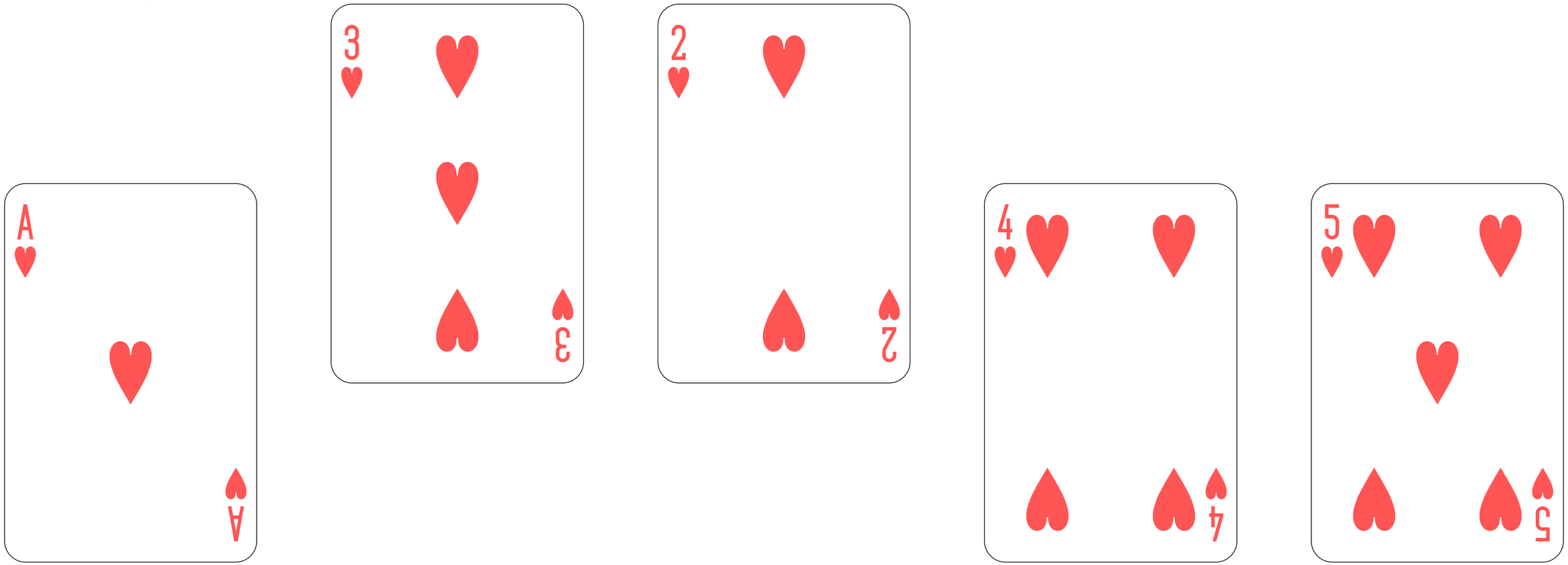

Sorting

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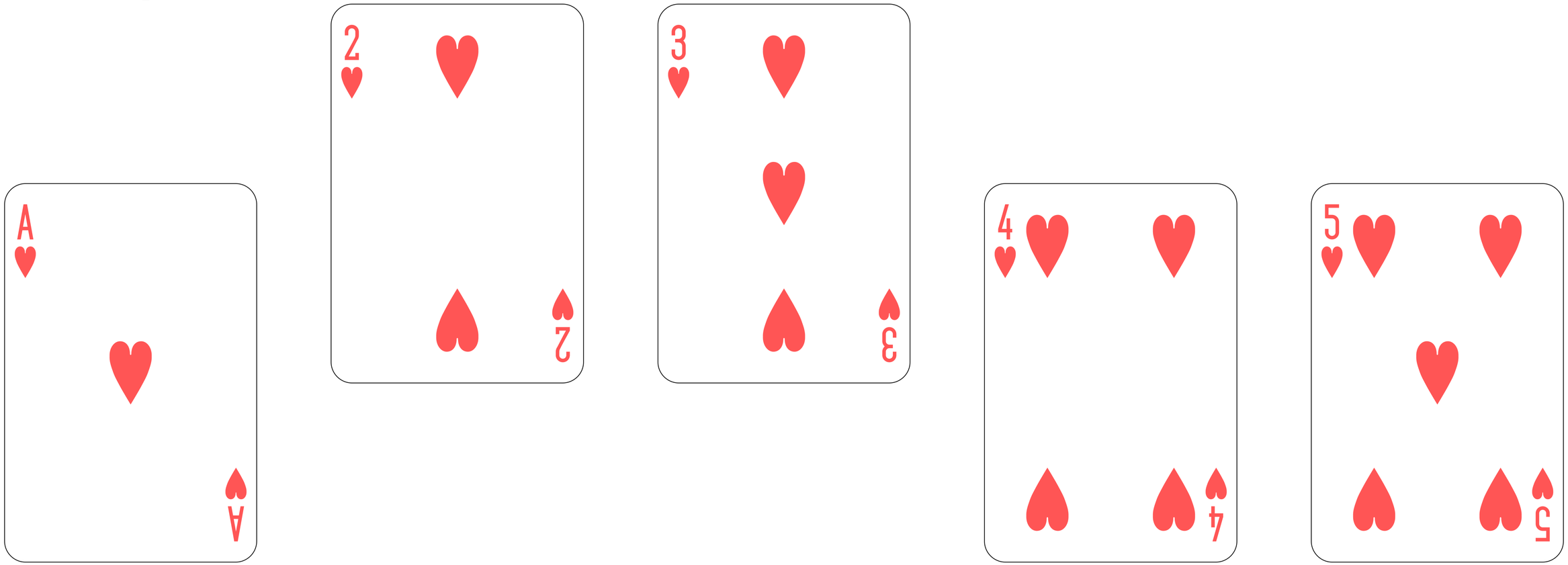

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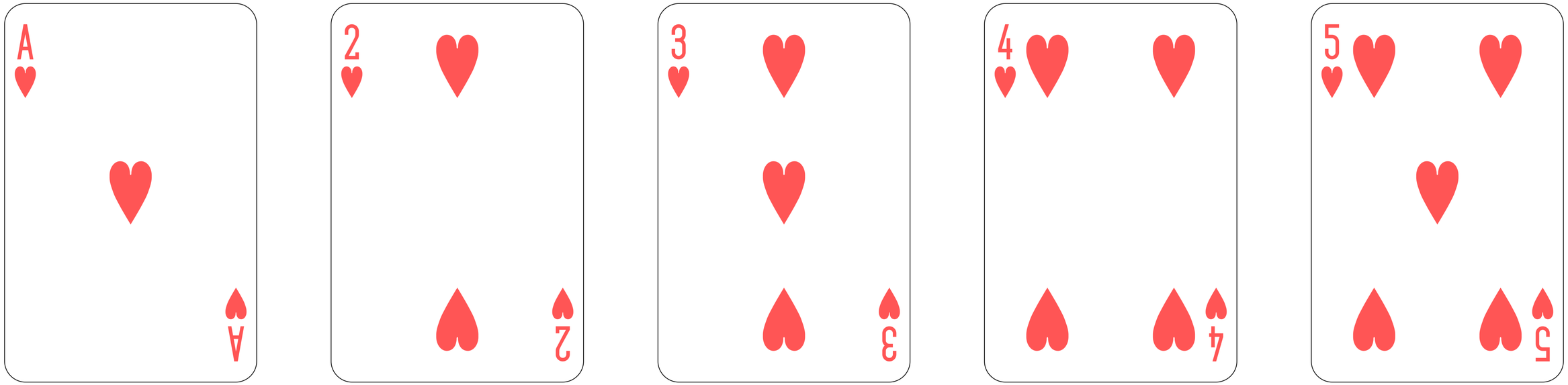

Sorting

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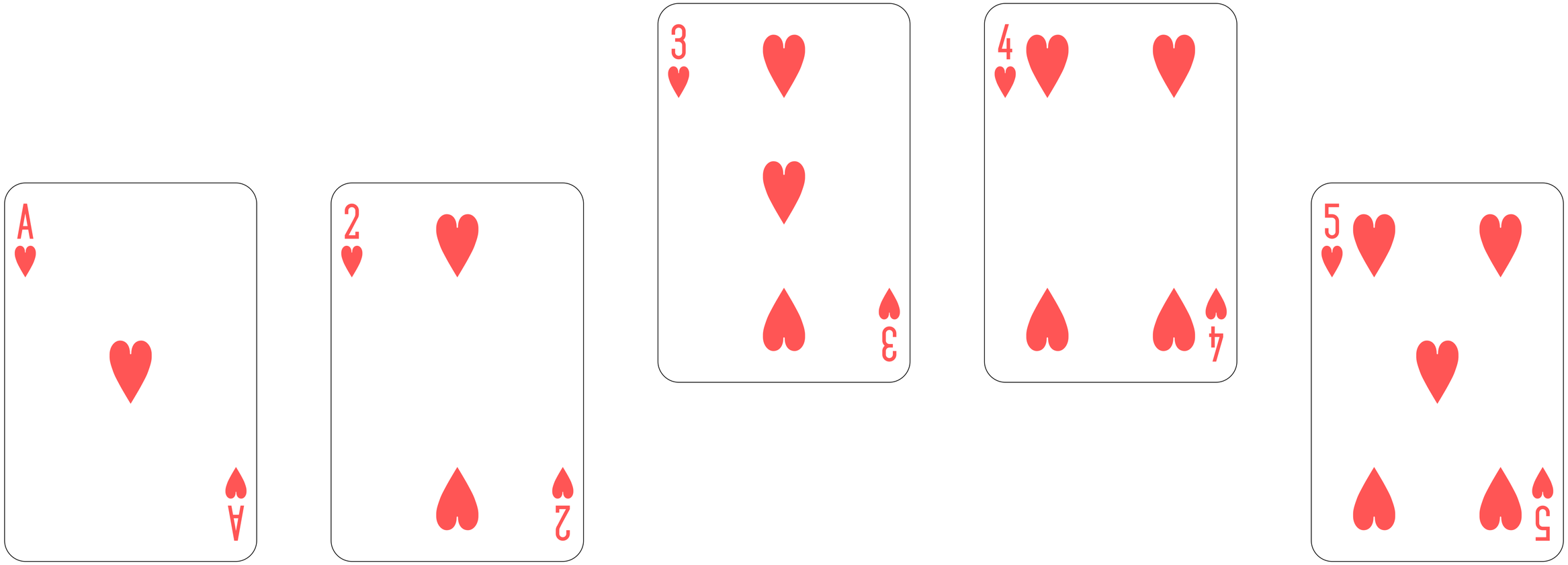

Sorting

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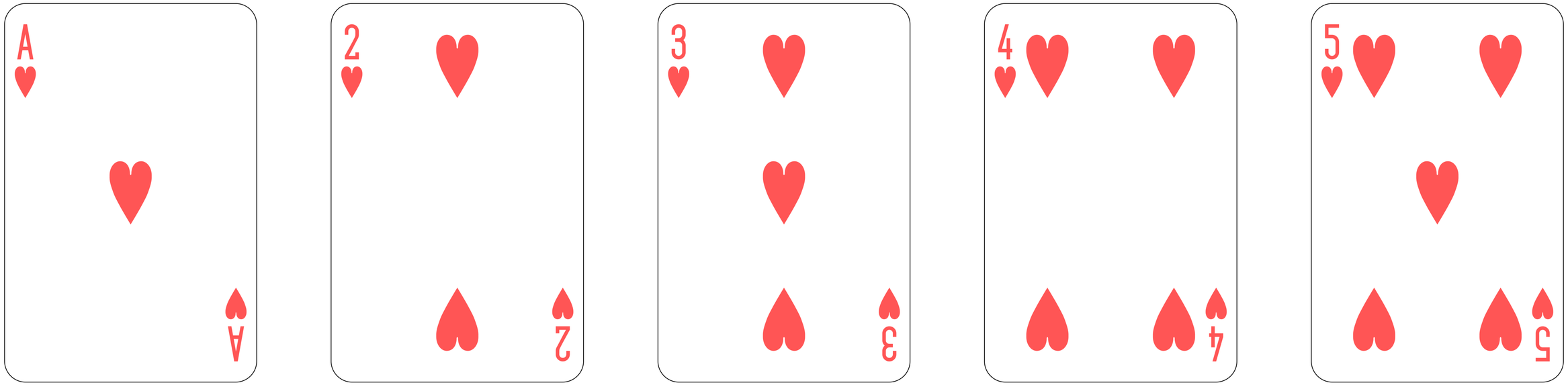

Sorting

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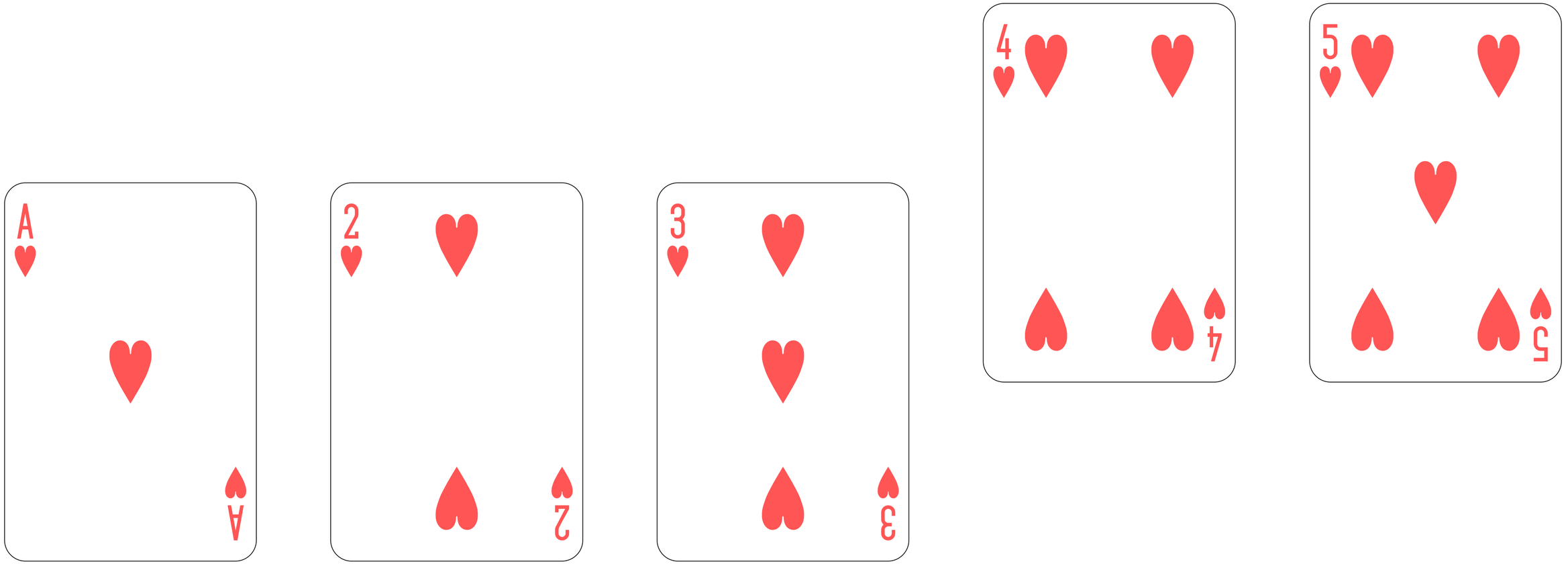

Sorting

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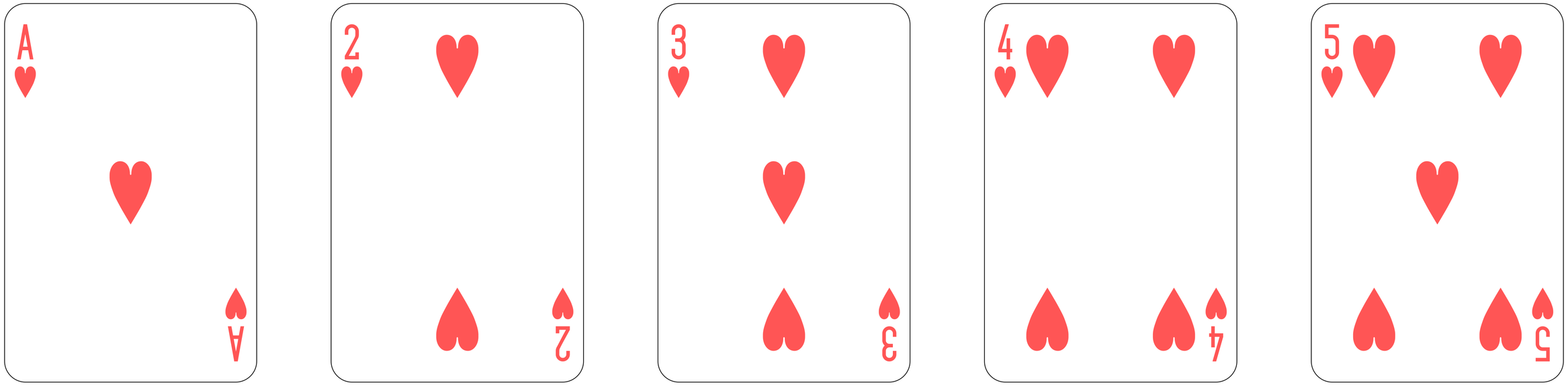

Sorting

Example 3: Algorithm

```
while True:
    swapped = False
    for i = 0 to n - 1:
        if A[i] > A[i + 1]:
            swap A[i], A[i + 1]
            swapped = True
    if not swapped:
        break
```

Sorting

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- Swap two adjacent elements if they are out of order

Sorting

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- How many rounds do we need to sort the entire list?

Sorting

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  - n. Why?

Sorting

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Sorting

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- Swap two adjacent elements if they are out of order
- How many rounds do we need to sort the entire list?
  - n. Why?
  - Every round, the largest element is pushed to the right.
- $O(n^2)$ comparisons, $O(n^2)$ swaps

Sorting

Example 3: Algorithm

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- The largest element "bubbles" up.

Sorting

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```

- The largest element "bubbles" up.
- This is called *bubble sort*.

Sorting

Sorting

- Three $O(n^2)$ algorithms: Insertion sort, selection sort, bubble sort

Sorting

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Sorting

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Sorting

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- It's a whole can of worms

Sorting

- Three $O(n^2)$
- There are be
  - We will re
- It's a whole

rt

Sort Benchmark Home Page

New: We are happy to announce the 2022 winners listed below. The new, 2022 records are listed in green. Congratulations to the winners!

Background

Until 2007, the sort benchmarks were primarily defined, sponsored and administered by Jim Gray. Following Jim's disappearance at sea in January 2007, the colleagues and sort benchmark winners. The Sort Benchmark committee members include:

- Chris Nyberg of Ordinal Technology Corp
- Mehul Shah of Aryn.ai
- George Porter of UC San Diego Computer Science & Engineering Dept

Top Results

	Daytona	Indy
Gray	2016, 44.8 TB/min **Tencent Sort** 100 TB in 134 Seconds 512 nodes x (2 OpenPOWER 10-core POWER8 2.926 GHz, 512 GB memory, 4x Huawei ES3600P V3 1.2TB NVMe SSD, 100Gb Mellanox ConnectX4-EN) Jie Jiang, Lixiong Zheng, Junfeng Pu, Xiong Cheng, Chongqing Zhao Tencent Corporation Mark R. Nutter, Jeremy D. Schaub	2016, 60.7 TB/min **Tencent Sort** 100 TB in 98.8 Seconds 512 nodes x (2 OpenPOWER 10-core POWER8 2.926 GHz, 512 GB memory, 4x Huawei ES3600P V3 1.2TB NVMe SSD, 100Gb Mellanox ConnectX4-EN) Jie Jiang, Lixiong Zheng, Junfeng Pu, Xiong Cheng, Chongqing Zhao Tencent Corporation Mark R. Nutter, Jeremy D. Schaub
Cloud	2016, $1.44 / TB **NADSort** 100 TB for $144 394 Alibaba Cloud ECS ecs.n1.large nodes x (Haswell E5-2680 v3, 8 GB memory, 40GB Ultra Cloud Disk, 4x 135GB SSD Cloud Disk) Qian Wang, Rong Gu, Yihua Huang Nanjing University Reynold Xin Databricks Inc. Wei Wu, Jun Song, Junluan Xia Alibaba Group Inc.	2022, $0.97 / TB **Exoshuffle-CloudSort** 100 TB for $97 40 Amazon EC2 i4i.4xlarge nodes 1 Amazon EC2 r6i.2xlarge node Amazon S3 storage Frank Sifei Luan UC Berkeley Stephanie Wang UC Berkeley and Anyscale Samyukta Yagati, Sean Kim, Kenneth Lien, Isaac Ong, Tony Hong UC Berkeley SangBin Cho, Eric Liang Anyscale Ion Stoica UC Berkeley and Anyscale
	2016, 37 TB	2016, 55 TB

Function Pointers

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- Your code lives in memory too!

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- ...so they have addresses

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- ...so they have addresses
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Function Pointers

- Your code lives in memory too!
- ...so they have addresses
- ...so just like we have pointers to data, we have pointers to functions as well
- What's the point?
  - We can pass functions around!

Function Pointers

Example

Function Pointers

Example

```
void alist_sort(struct alist *l, int (*cmp)(void *, void *))
```

Function Pointers

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- The second argument to this function is

Function Pointers

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Function Pointers

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void alist_sort(struct alist *l, int (*cmp)(void *, void *))
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- The second argument to this function is
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Function Pointers

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  - It tells the sorting function how to compare two arbitrary elements

Function Pointers

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- The second argument to this function is
  - A function pointer called `cmp`
  - The function that `cmp` points to takes two `void *` and returns `int`
  - It tells the sorting function how to compare two arbitrary elements
  - (negative if 1 < 2, 0 if 1 == 2, positive if 1 > 2)

Function Pointers

Example

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Function Pointers

Example

```
void alist_sort(struct alist *l, int (*cmp)(void *, void *)) {
    for (;;) {
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Function Pointers

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void alist_sort(struct alist *l, int (*cmp)(void *, void *)) {
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        int swapped = 0;
```

Function Pointers

Example

```
void alist_sort(struct alist *l, int (*cmp)(void *, void *)) {
    for (;;) {
        int swapped = 0;
        for (int i = 0; i < l->length; i++) {
```

Function Pointers

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void alist_sort(struct alist *l, int (*cmp)(void *, void *)) {
    for (;;) {
        int swapped = 0;
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            if (cmp(l->elems[i], l->elems[i + 1]) < 0) {
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Function Pointers

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                swapped = 1;
            }
        }
```

Function Pointers

Example

```
void alist_sort(struct alist *l, int (*cmp)(void *, void *)) {
    for (;;) {
        int swapped = 0;
        for (int i = 0; i < l->length; i++) {
            if (cmp(l->elems[i], l->elems[i + 1]) < 0) {
                void *tmp = l->elems[i];
                l->elems[i] = l->elems[i + 1];
                l->elems[i + 1] = tmp;
                swapped = 1;
            }
        }

        if (!swapped) {
```

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        if (!swapped) {
            break;
```

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Function Pointers

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            }
        }

        if (!swapped) {
            break;
        }
    }
}
```

Function Pointers

Example

```
void alist_sort(struct alist *l, int (*cmp)(void *, void *));

int strcmp_wrapper(void *s1, void *s2) {
    return strcmp(s1, s2);
}

int main(void) {
    struct alist l;
    alist_sort(&l, &strcmp_wrapper);
    return 0;
}
```

Function Pointers

Example

```
void alist_sort(struct alist *l, int (*cmp)(void *, void *));

int strcmp_wrapper(void *s1, void *s2) {
    return strcmp(s1, s2);
}

int main(void) {
    struct alist l;
    alist_sort(&l, &strcmp_wrapper);
    return 0;
}
```

^ optional