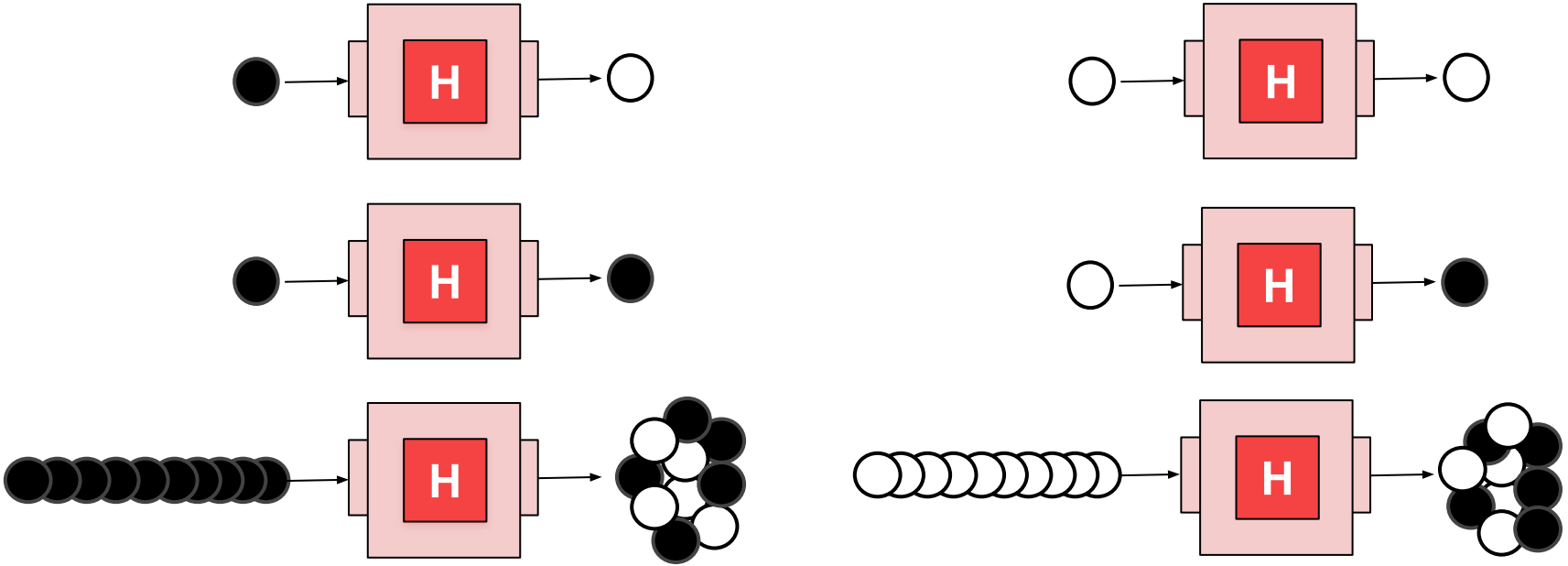
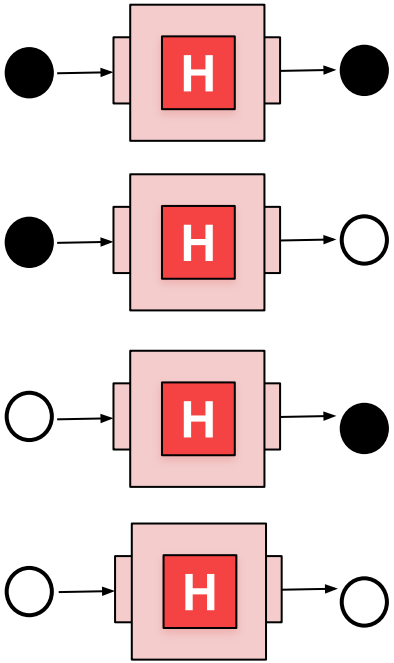
Probabilistic Gate: H

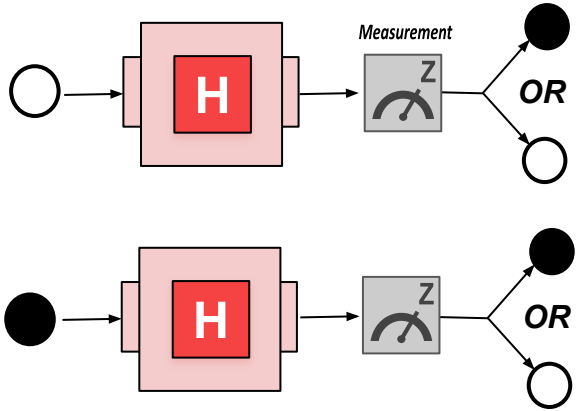
New Gate: The **H** Gate



New Gate: The **H** Gate

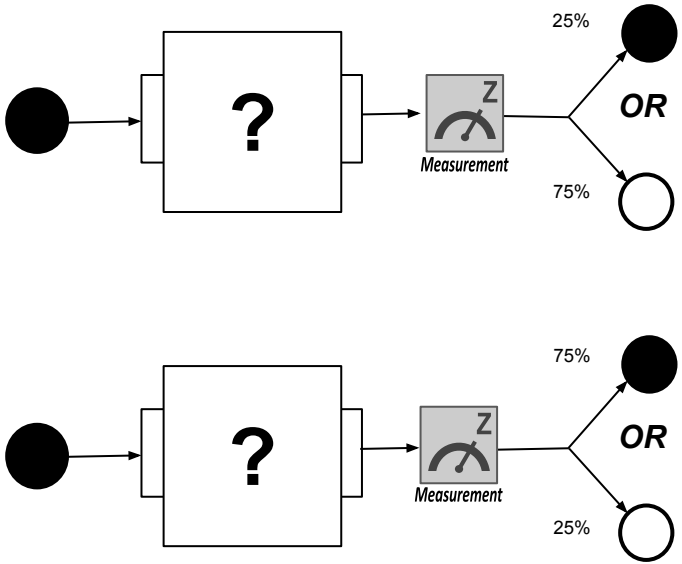


Observed outcome from single gate is random!



Probability (50%), not outcome, is predictable.

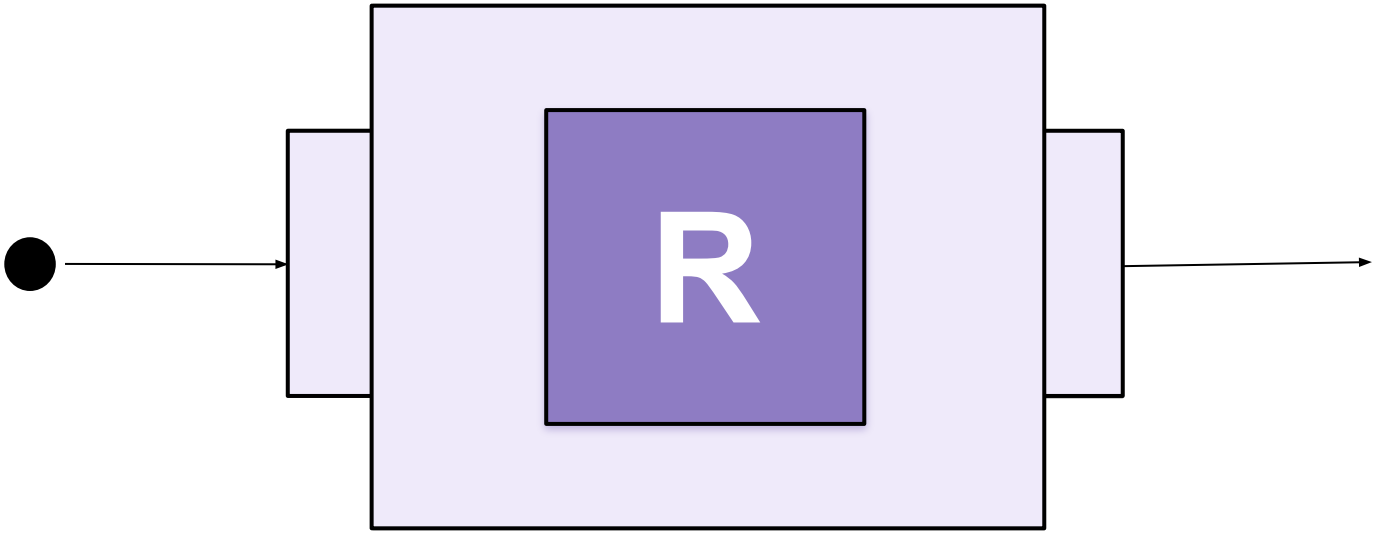
Gates exist that result in the following



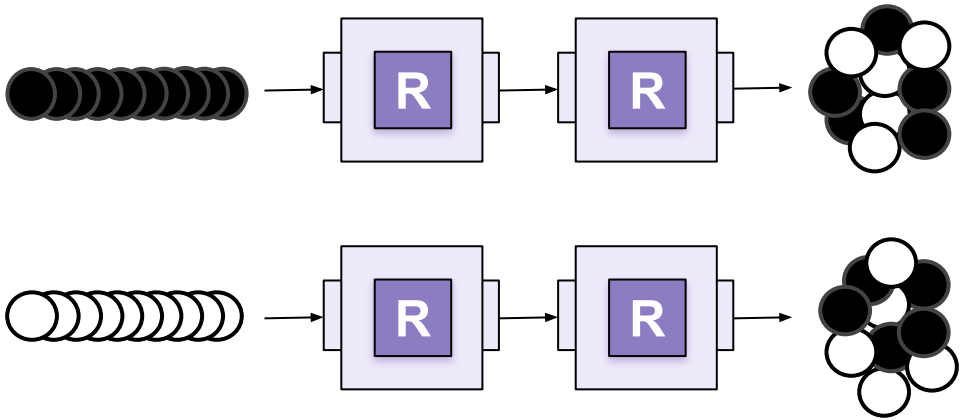
Learning Goals

- ➔ What is going on with these probabilistic gates?
- ➔ How do we represent the probabilistic output visually?
- ➔ How do we represent the probabilistic output mathematically?
- ➔ How can we calculate these probabilities as they pass through other gates?

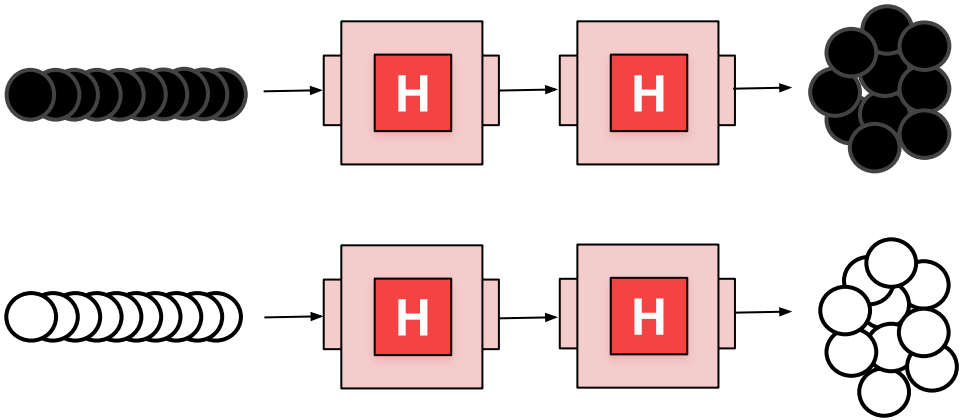
Is the H gate a *random* gate? How would we know?



If it is an internal coin flip, what is the expected outcome?

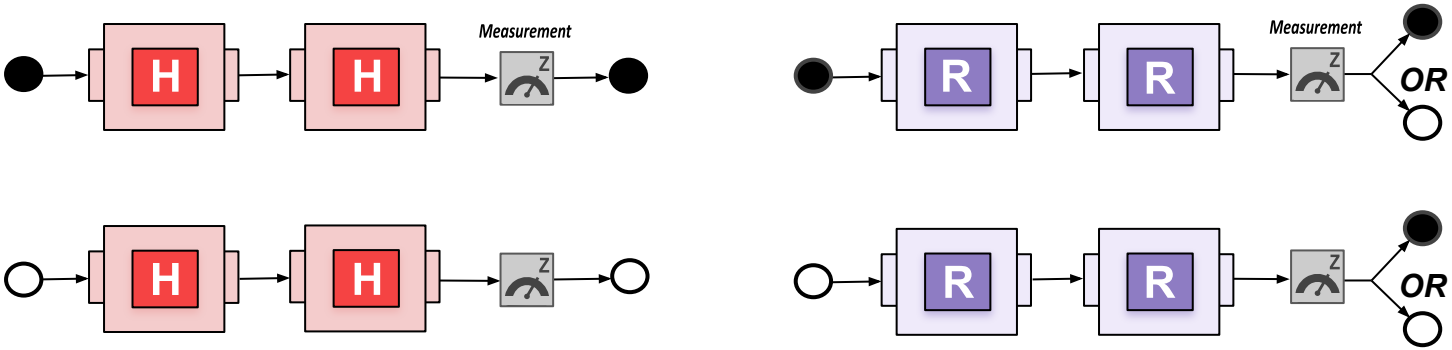


Here is what actually happens...



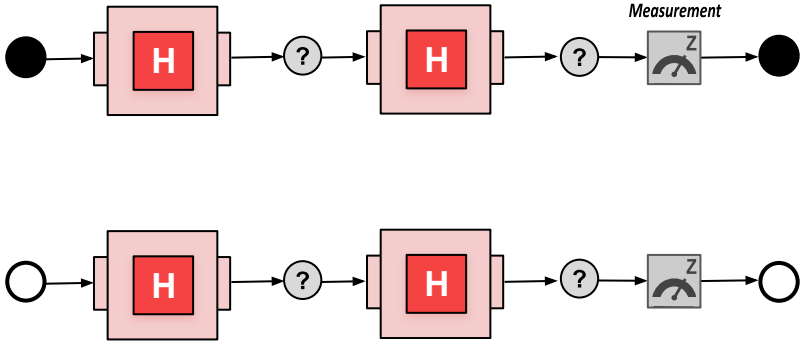
Deduction: **H** gate ***appears*** random, but it is not!

What makes the **H** *gate different from a purely random gate* (**R**)*?*

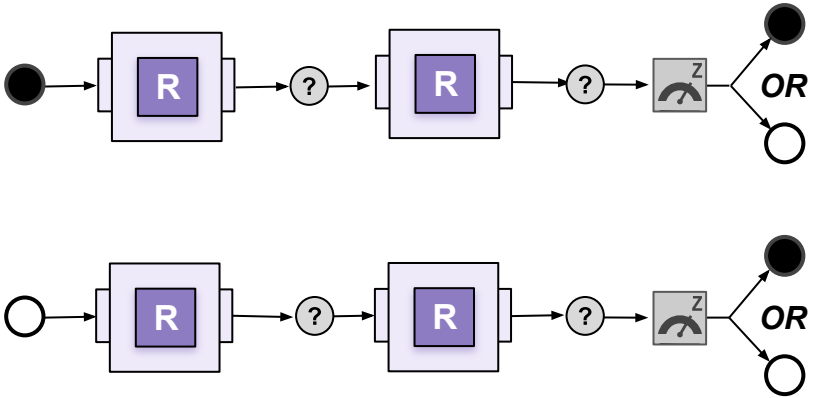


Two instances ***always*** brings it back to the initial color.
Therefore, it cannot be random.

Deductions:



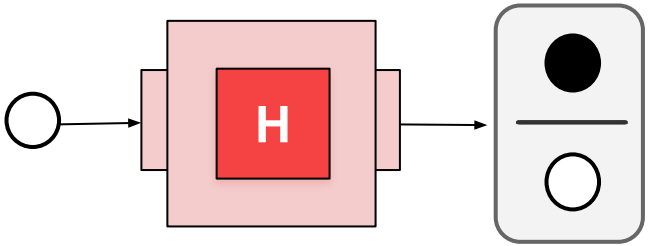
A black ball's output is different from a white ball's output.



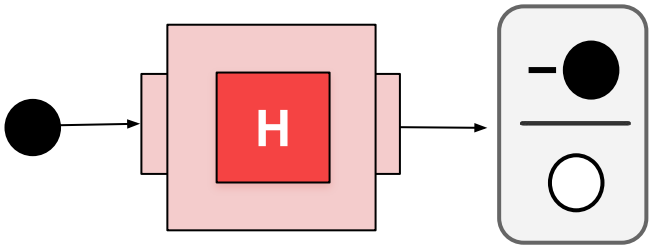
After an **H** gate, but before measurement, the ball is not simply black or white, but something more complex.

Superposition state:

Equal probability of each outcome.



The negative sign (-) indicates an additional aspect of state.



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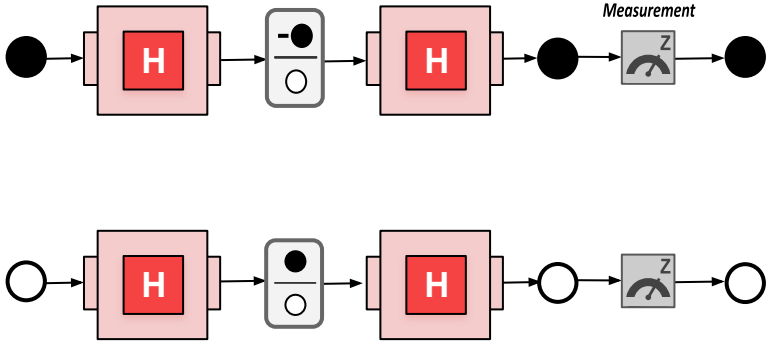

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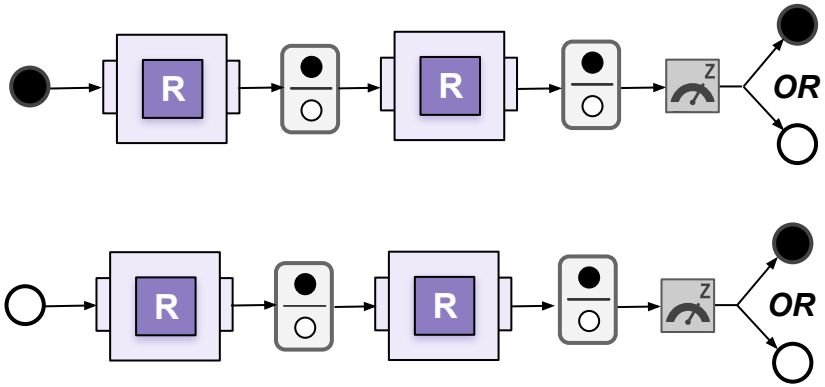

After an **H** gate, but before measurement, the ball is not simply black or white, but something more complex.

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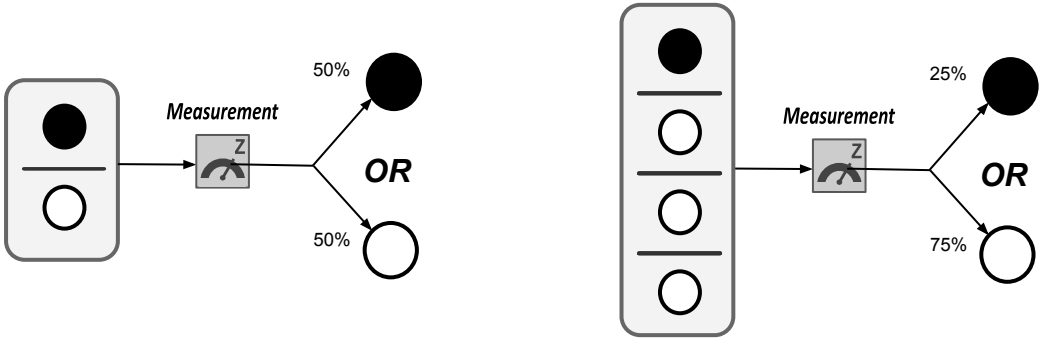
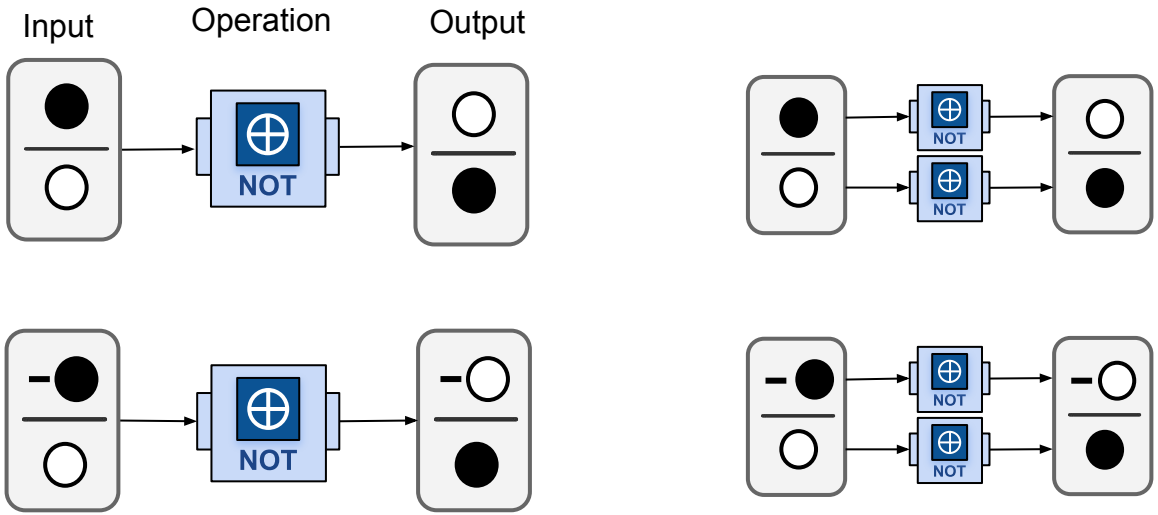
Visual Superposition State

Beyond 50/50 Superposition

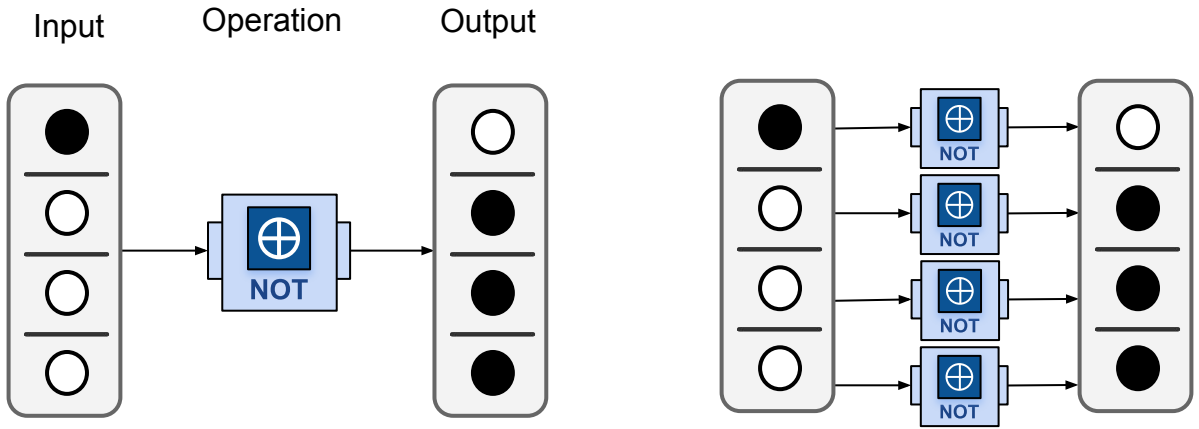


Superposition as input:



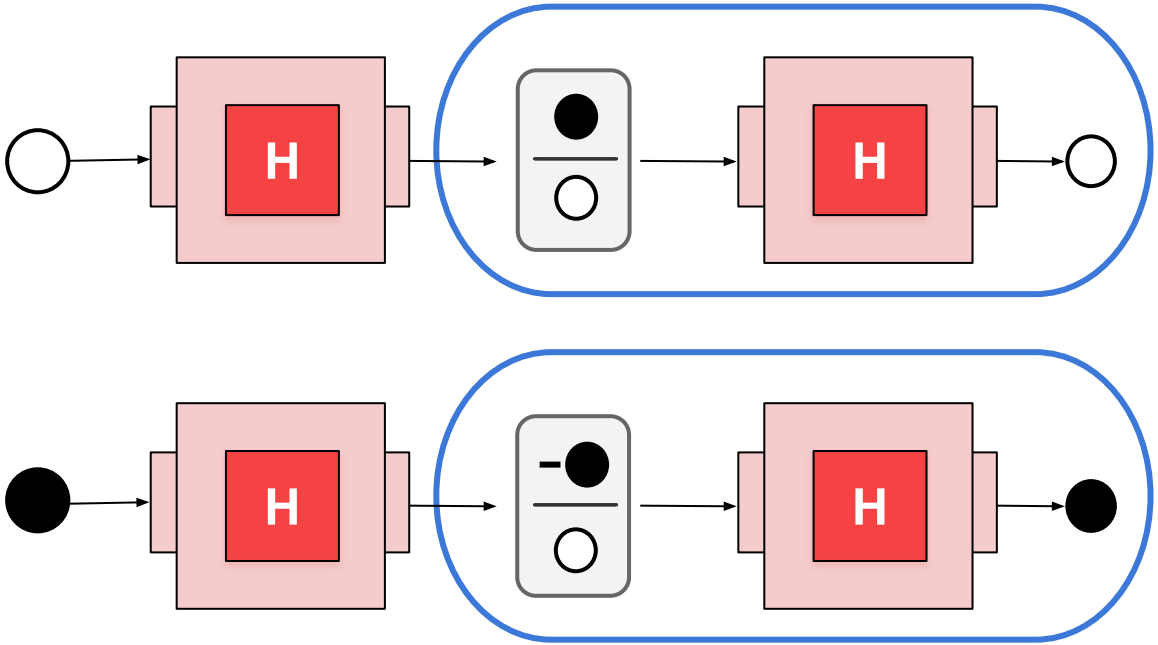
Apply a **NOT** gate to each

Superposition as input:

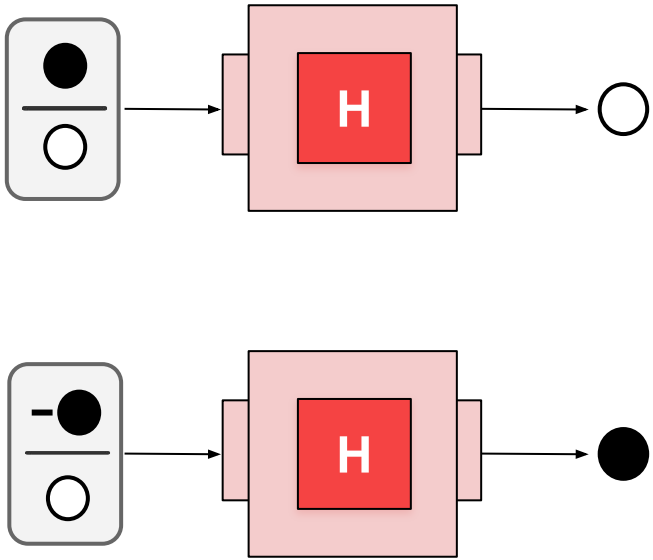


Apply a **NOT** gate to each

Revisiting odd behavior...



Revisiting odd behavior...



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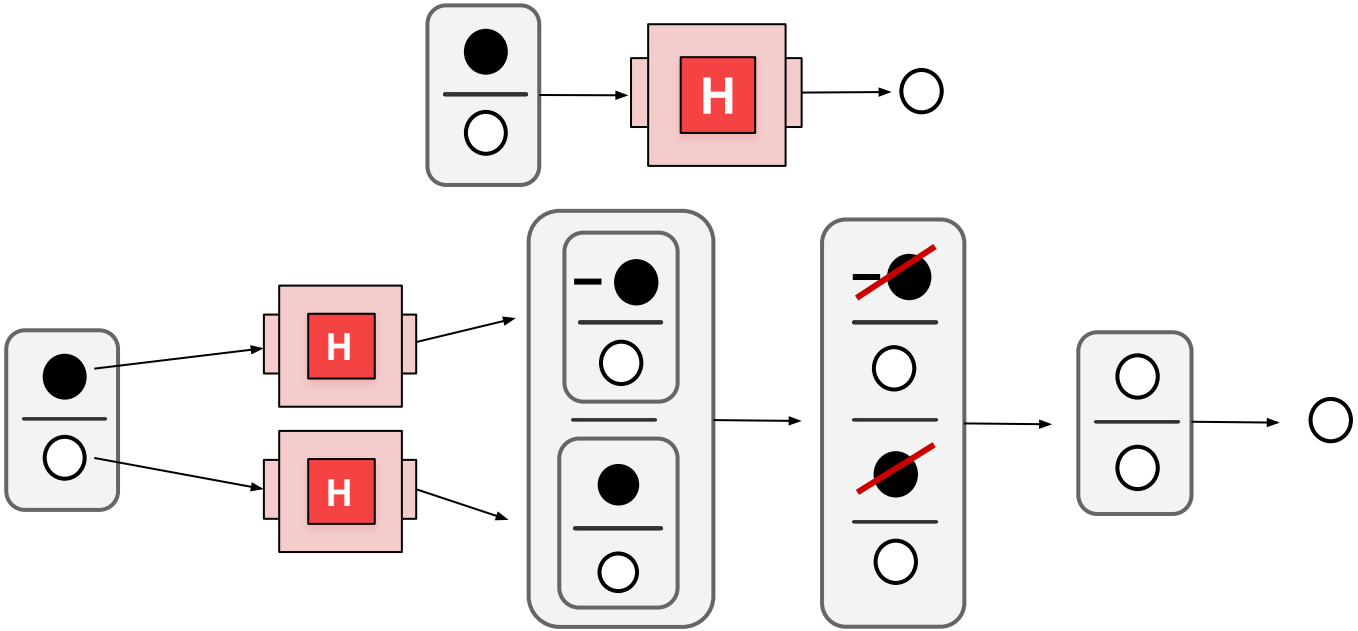
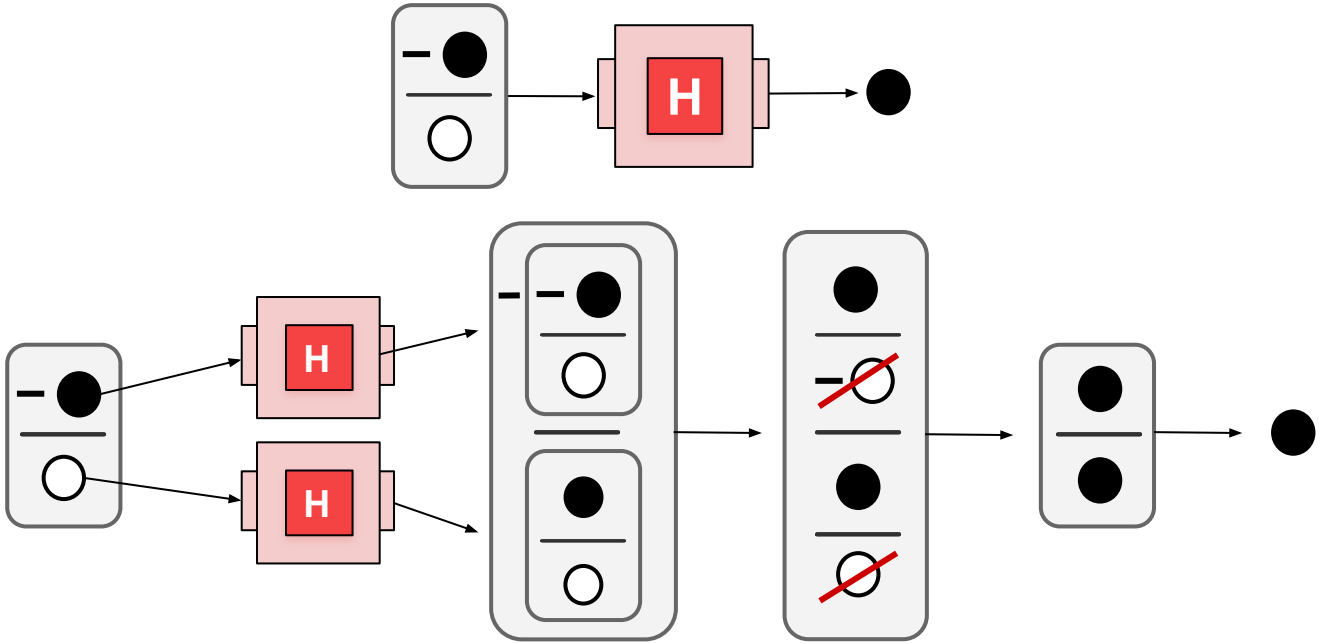


Revisiting odd behavior...



Summary

- ➔ The **H** gate puts a qubit in superposition.
- ➔ An **H** gate applied to a state of $|0\rangle$ or $|1\rangle$ results in 50/50 chance of measuring 0 or 1.
- ➔ Two **H** gates in sequence reverse each other, resulting in the original input.
- ➔ Thus, there is more to state than just the probability of measuring 0 or 1 - there is also ***phase***.
- ➔ Our calculation with the phase value accurately models / predicts this reversing behavior.

Introduction to Probability

What does the probability of rain really mean?!?

What can we conclude from this weather forecast?

It is not going to rain at 11pm?

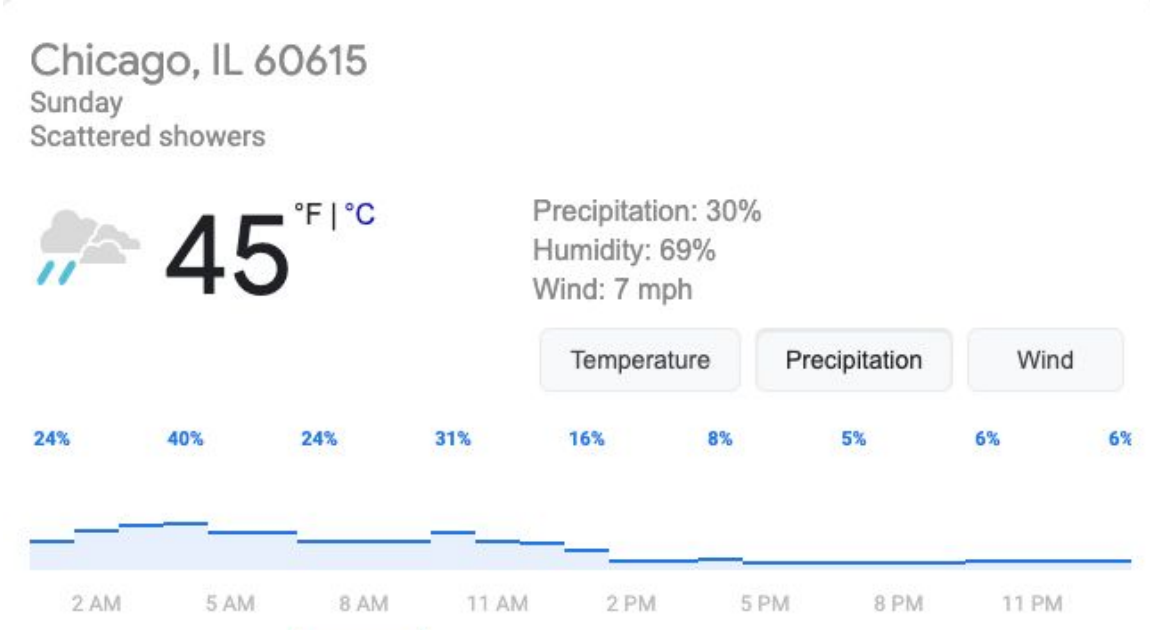
Not a valid conclusion

It is going to rain at 4am?

Not a valid conclusion

It is more likely to rain at 8am than 8pm?

Valid conclusion

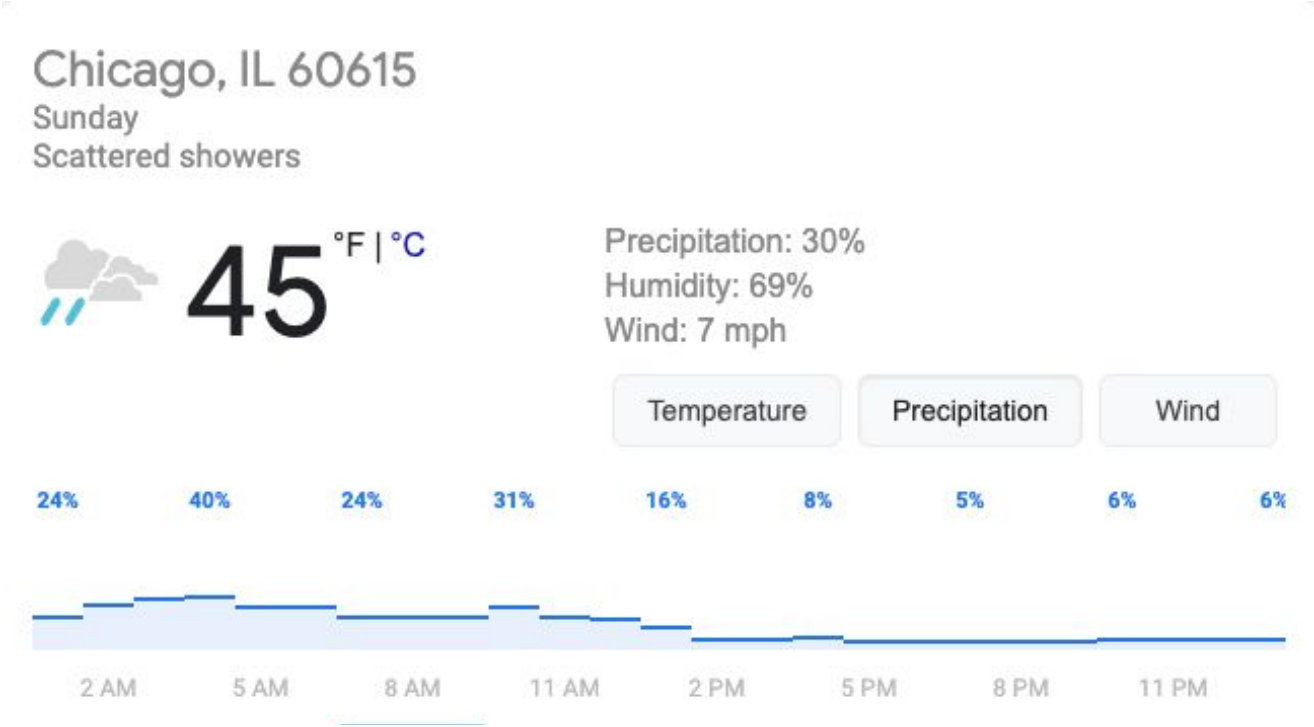


It does not tell us whether or not it will rain on Sunday.

What does the probability of rain really mean?!?

Given our limited information:

If we experienced 100,000 days with identical conditions, approximately 40,000 of them would have rain at 4am.



Probability is **not**….

A prediction about the outcome of any individual action

Confirmed or refuted through a single experiment

Probability is….

A prediction of the frequency of an outcome of many, many actions

Confirmed or refuted through many, many experiments

Combining multiple independent events

There is a 50% chance (probability) of rain in the afternoon.

There is a 25% chance (probability) I will remember my raincoat.

What is the probability that it will both rain **and** I will forget my raincoat?

This depicts the independent probabilities

75% or ¾

25% or ¼

50% or ½

50% or ½

To combine them, we can make combinations

50% or ½

75% or ¾

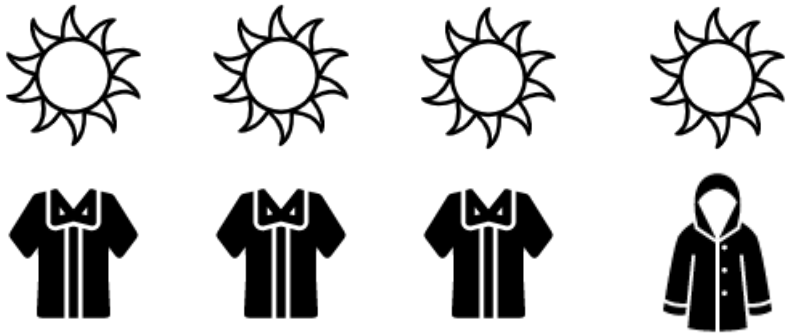
25% or ¼

50% or ½

75% or ¾

25% or ¼

8 equally-likely, non-unique scenarios



It rained and I forgot my raincoat

⅜ or 37.5%

Or…. calculate to get each unique outcome

75% or 0.75

25% or 0.25

50% or 0.5

50% or 0.5

0.75 0.5

0.75 * 0.5 = 0.375 = 37.5%

0.25 0.5

0.25 * 0.5 = 0.125 = 12.5%

0.75 0.5

0.75 * 0.5 = 0.375 = 37.5%

0.25 0.5

0.25 * 0.5 = 0.125 = 12.5%

How is probability used in Quantum Computing?

Each qubit in superposition has a probability of being measured 0 or 1.

Multiple qubits are required in order to perform useful computation.

Qubits start with independent probabilities, but then they become multi-qubit combinations.

Random Thoughts...

*Or thoughts about the word **random***

That was such a ***random*** comment!

That was ***unpredictable*** - to me!

Draw a number at ***random***!

Each number has ***equal probability*** of being drawn.

Quantum measurements have ***random*** outcomes!

We know the ***probability*** of an outcome, but the outcome of a single measurement is ***not guaranteed*** (indeterminate or nondeterministic)

Superposition & Measurement

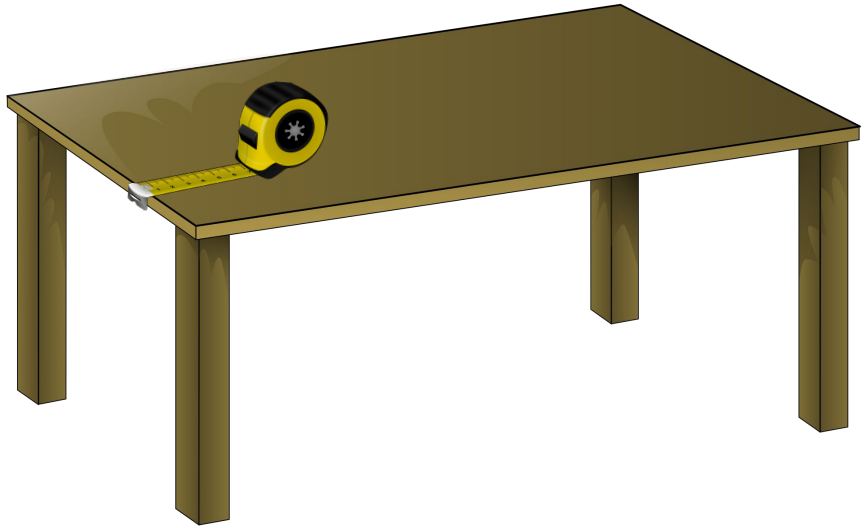
Measurement

Consists of a ***question***, a ***device,*** and a ***method***

Question: What is the table length?

Device: Tape Measure

Method:
Pull out the tape measure along the length of the table and read out the number at the end.



Measurement

Consists of a ***question***, a ***device*** and a ***method***

Question: What is this baby feeling?

Device: Eyes and ears

Method:
Look for smile, frown, or tears.
Listen for laughs, silence, or screams.

- Happy
- Excited
- Proud

- Hungry
- Frustrated
- Tired

Some measurements give only partial information!

Measurement

Consists of a ***question***, a ***device*** and a ***method***

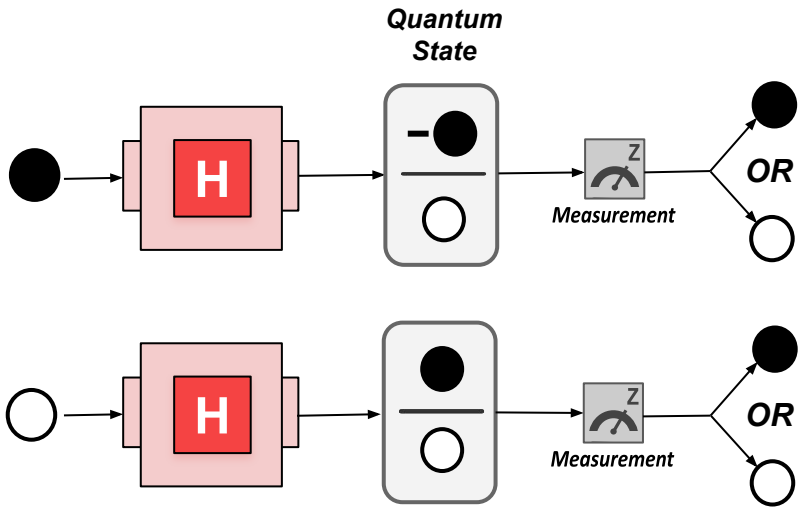
Question: How long can you hold your breath?

Device: Stopwatch

Method:
Use a stopwatch to time how long you can hold your breath.

Some measurements affect the item being measured!

Quantum Measurement



Measurement does not reveal full state!
Measurement reveals neither phase nor probabilities involved!

Superposition

- A single object can be multiple things at once
- State is suspended as a combination of multiple values

Measurement resolves a superposition

Question: Which definition of **polish** is this?

Need a **device** and **method**.

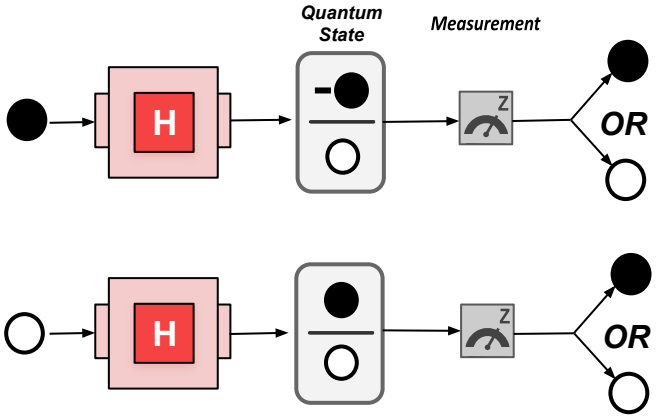
Your **ear** **hears** the word said aloud.

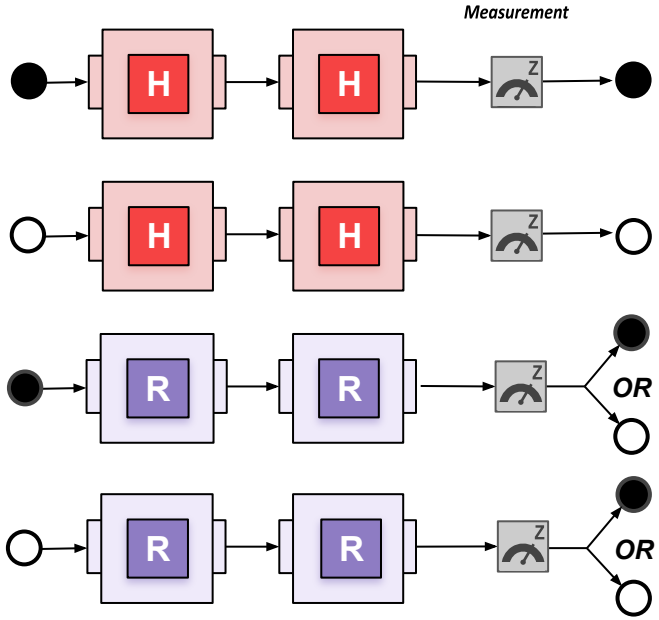
- "**Paw**-lish" \ ˈ**pä**-lish \
- "**Poe**-lish" \ ˈ**pō**-lish \

Your **eyes** **read** the words around it for context.

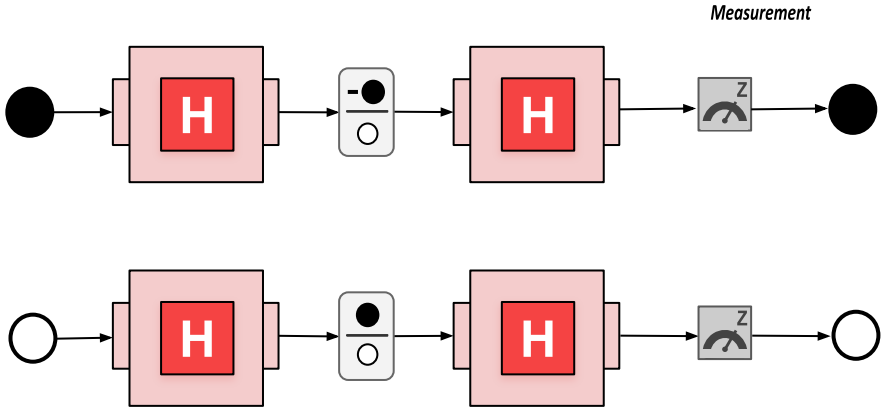
- "**Polish** makes the floor shine."
- "**Polish** sausage is delicious."

Revisiting H Gate:
The Role of Measurement



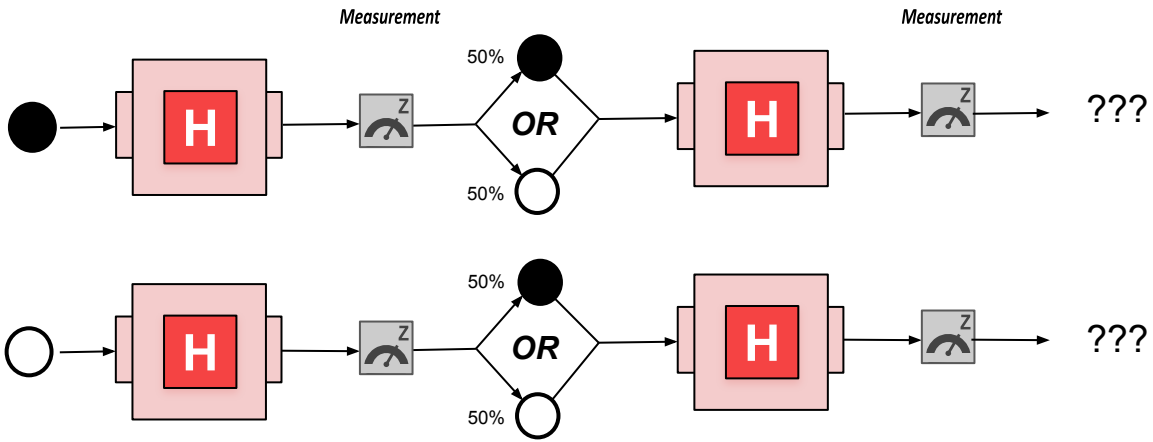


Predictable outcome w/out intermediate measurement

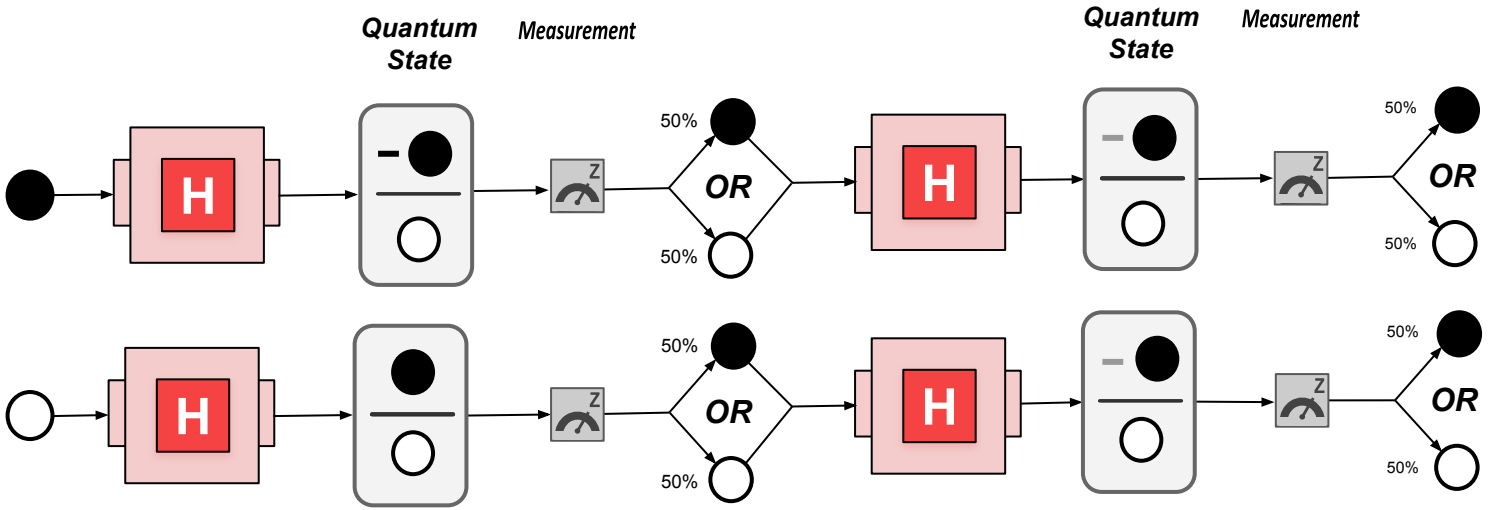


Superposition of complex state enters second gate

What happens if we try to observe the balls in between...



Unpredictable outcome with intermediate measurement



Measurement collapses superposition

Quantum Superposition

- ➔ A qubit is a superposition of two values: $|0\rangle$ or $|1\rangle$
- ➔ Part of quantum state is the ***probability*** of measuring 0 or 1
- ➔ The ***probability*** that a measurement detects one or the other can be ***manipulated*** through quantum operations
- ➔ ***Measurement*** cannot detect the entire state, only an individual 0 or 1
- ➔ The act of ***measurement collapses the superposition***, making the qubit become only the measured value 0 or 1