Hash Table (cont.)

CS143: lecture 17

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Hash Table

Recap

- Nice $O(1)$ complexity because we can index into an array instead of chasing pointers
- We have a way to turn anything into an integer -- hash function
- We have a way to force any integers into a reasonable range -- compression (usually modulus)
- We need to handle collisions:
    - Collisions can be the result of the hash function
    - ... of compression

Hash Table

Handling Collision

- Two approaches:
  1. Chaining
  2. Probing

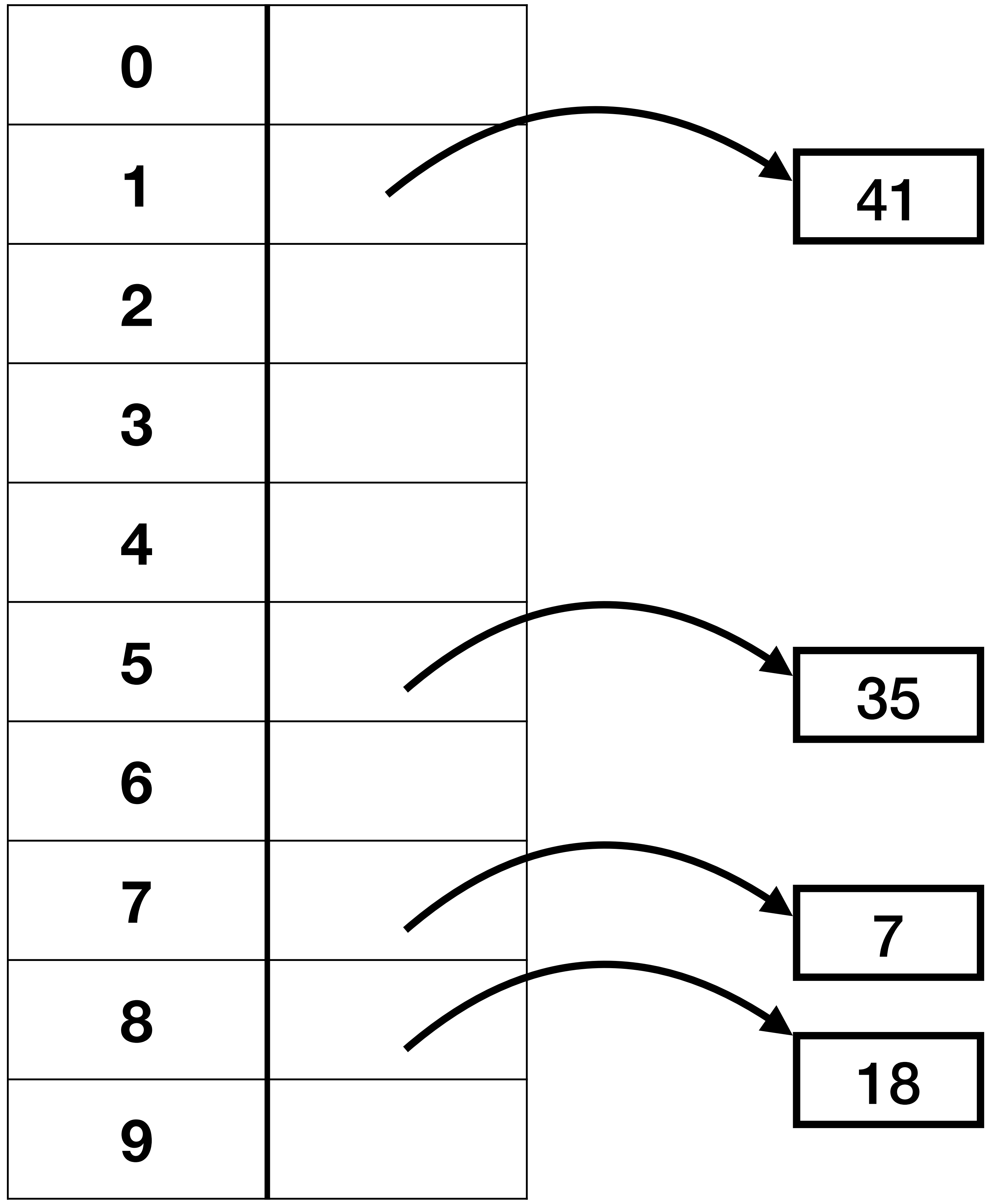
Chaining

- Each slot is a *list* of key-value pairs, called a *bucket*

0	
1	41
2	
3	
4	
5	35
6	
7	7
8	18
9	

Chaining

- Each slot is a *list* of key-value pairs, called a *bucket*

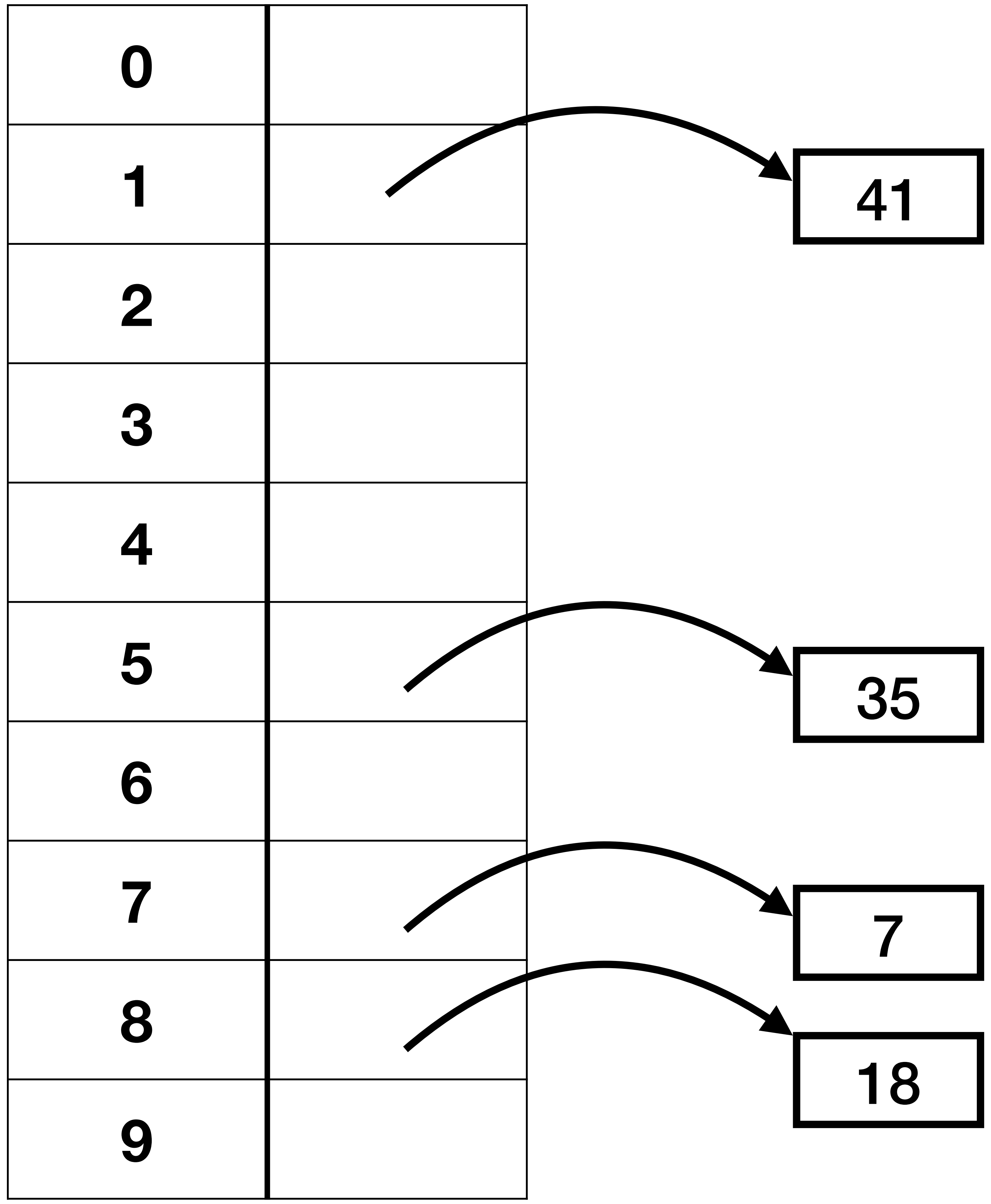


- You can use either list implementation
  - ...but there is an obvious choice
  - linked list, because of deletion

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*

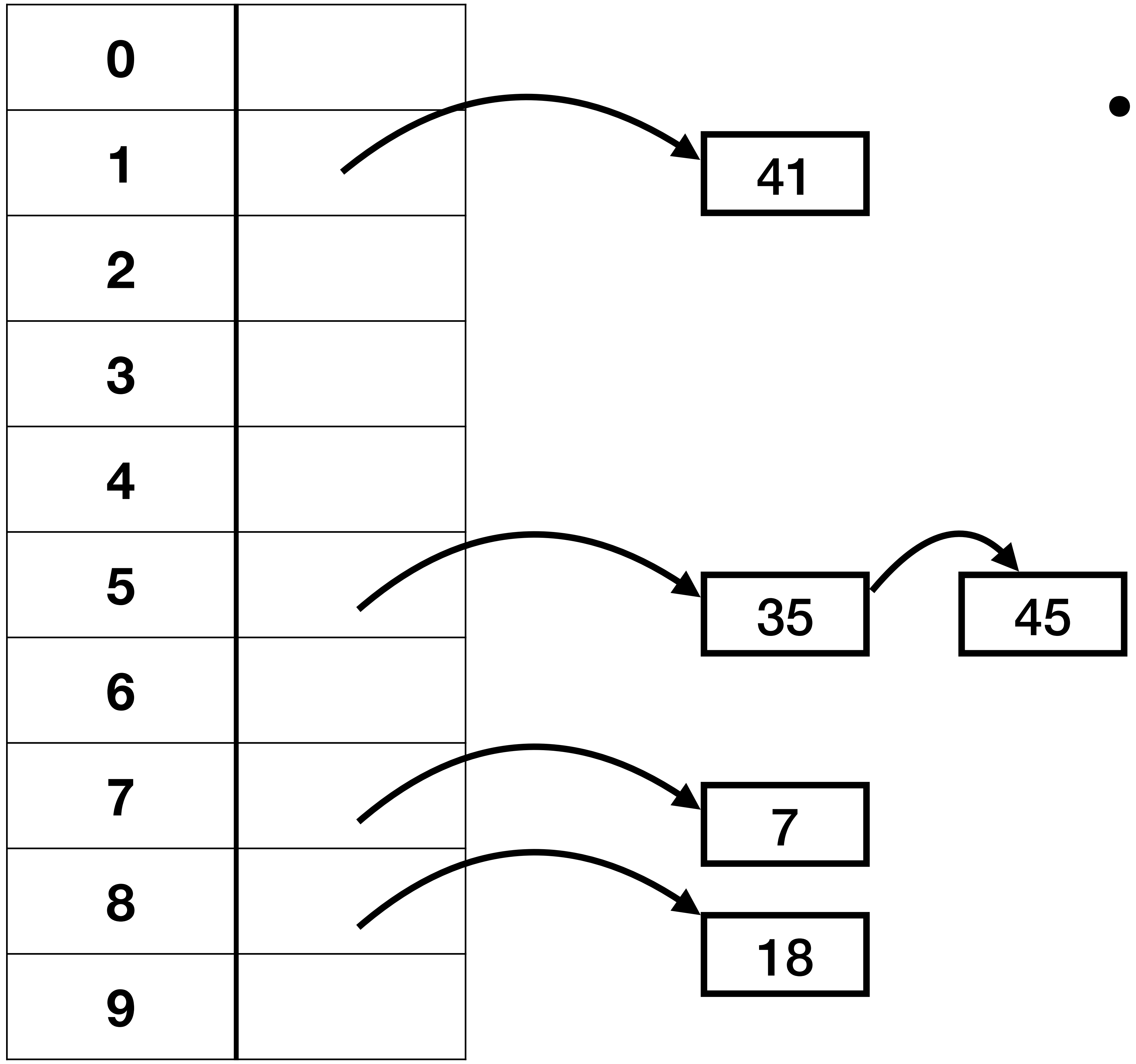


- Collisions will be prepended into the list

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*

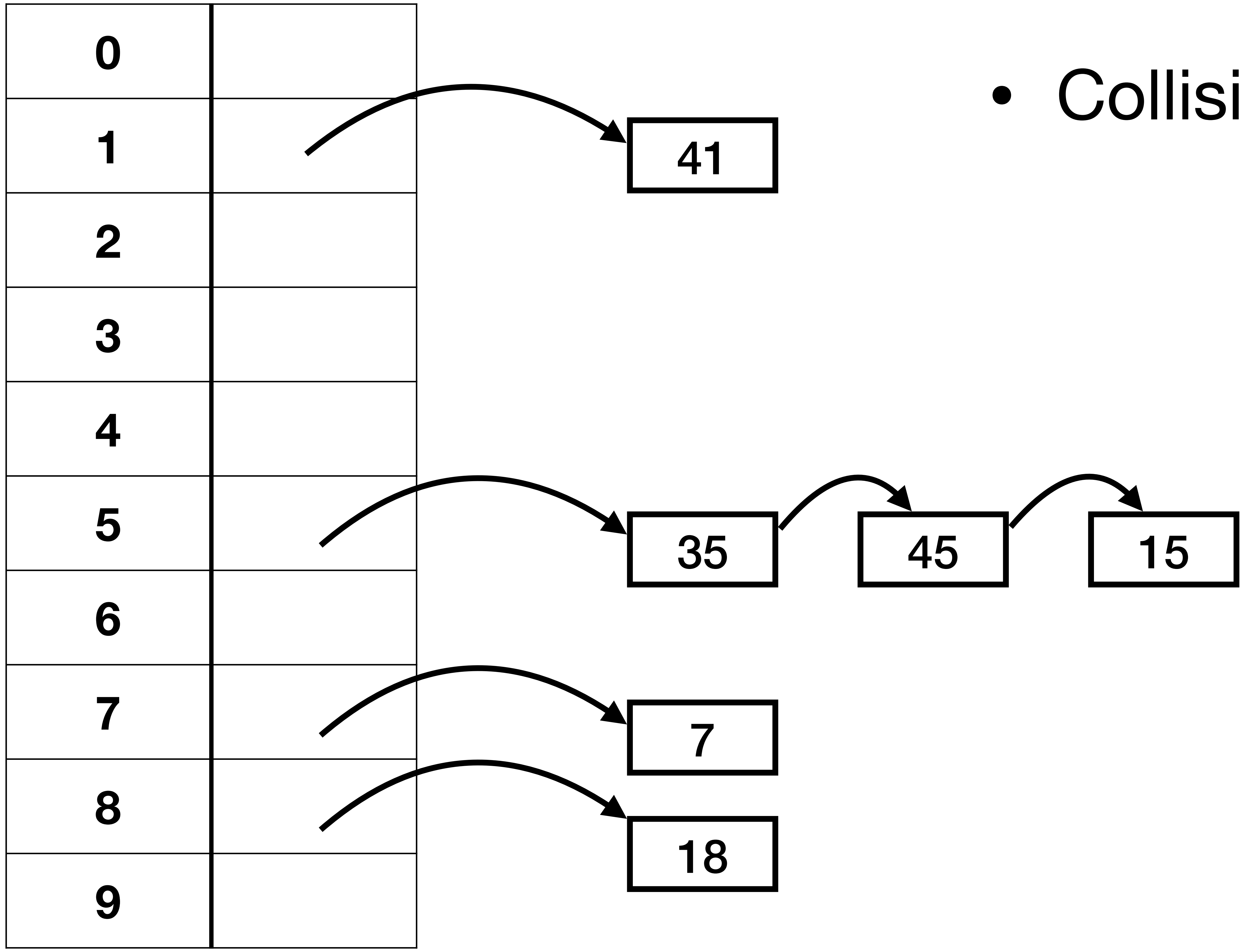


- Collisions will be prepended into the list

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*

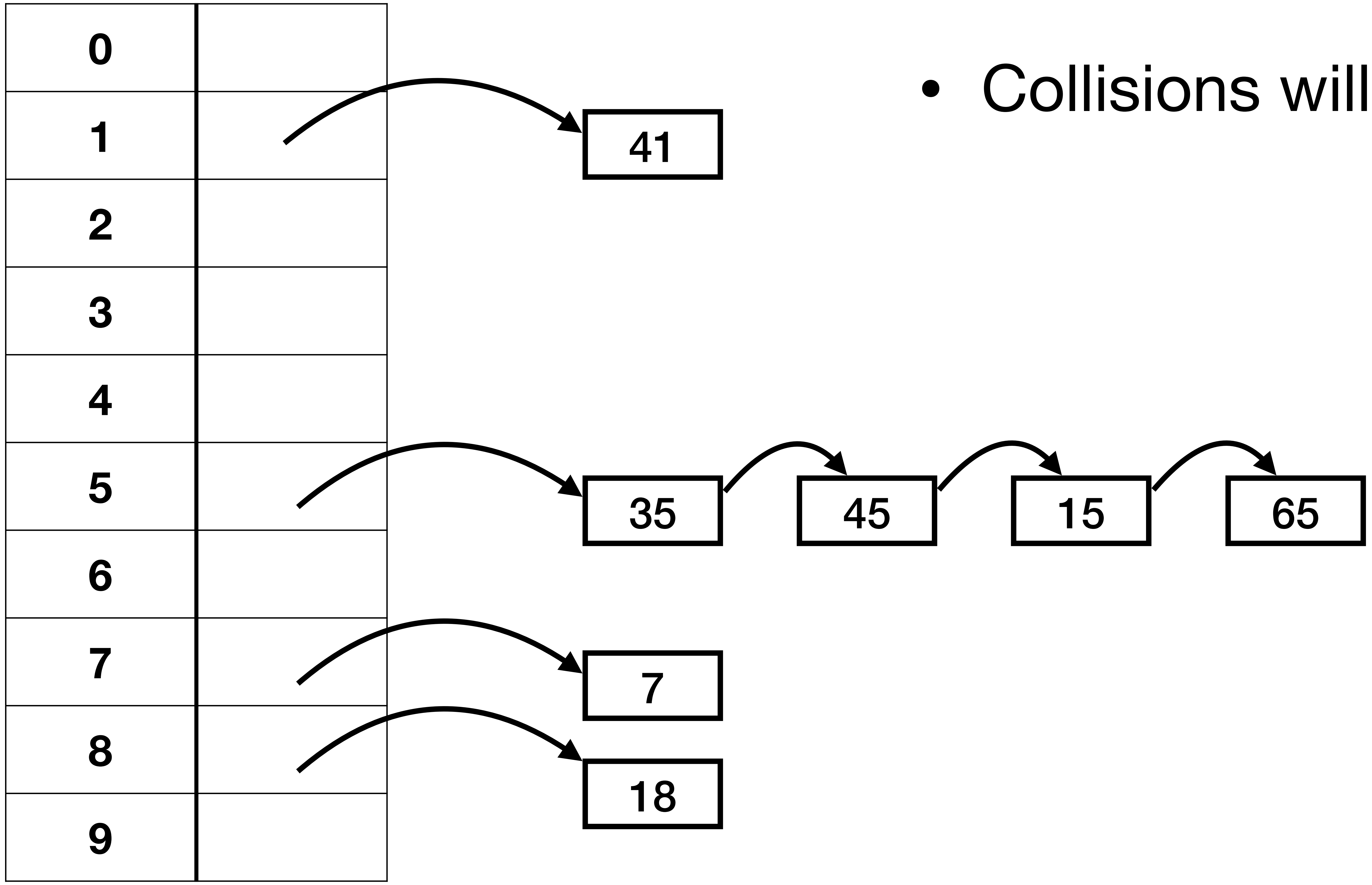


- Collisions will be prepended into the list

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*

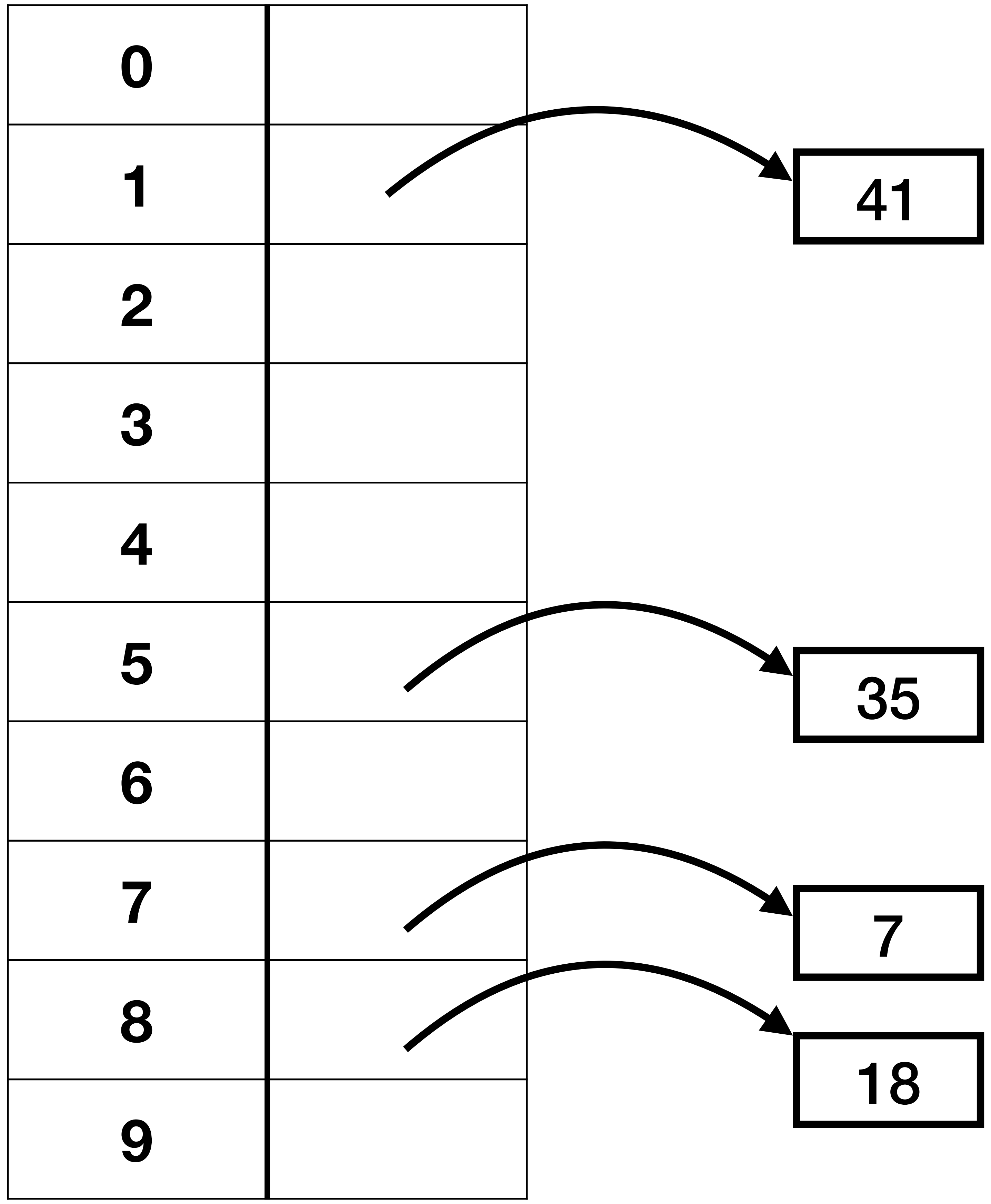


- Collisions will be prepended into the list

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*

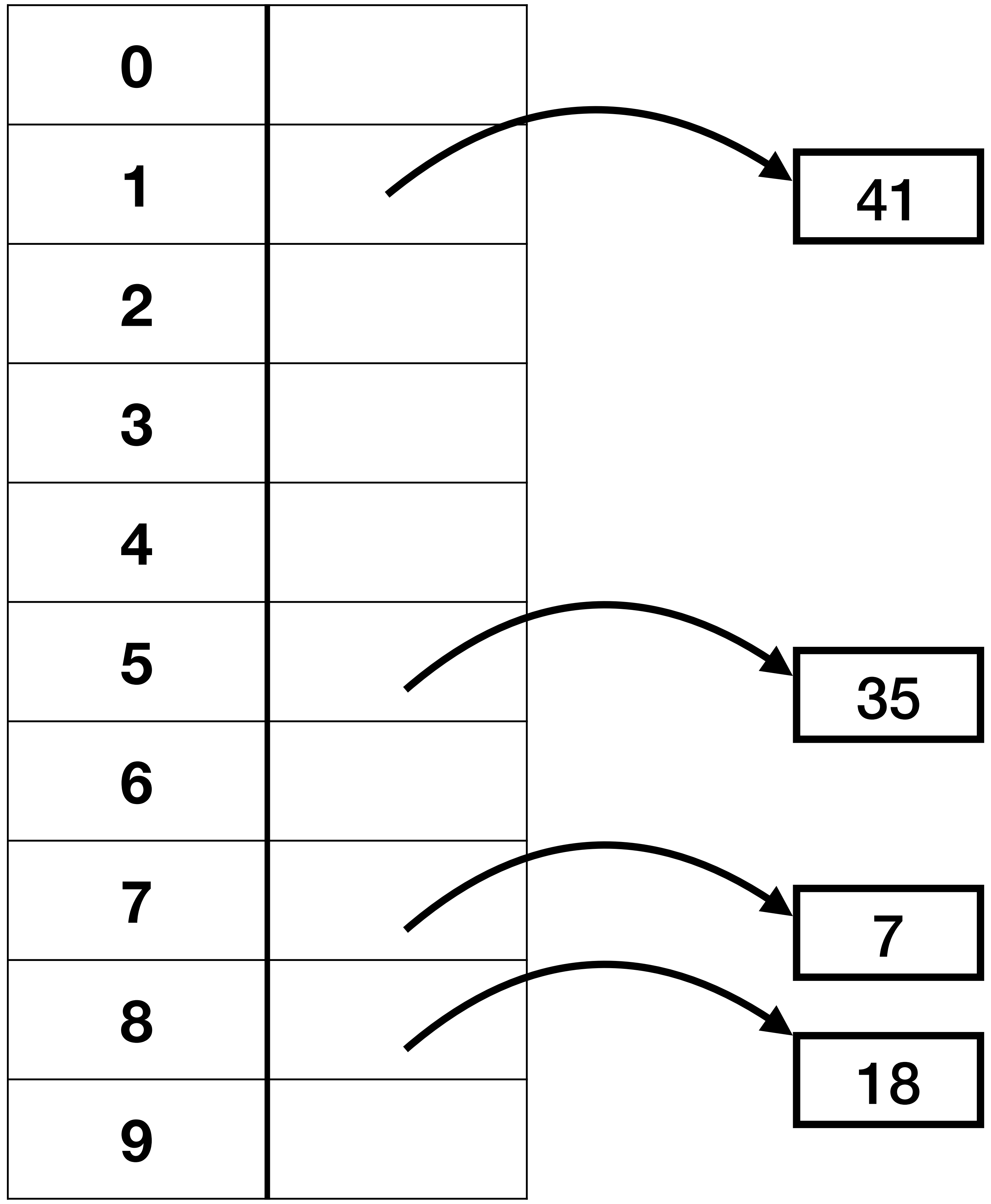



```
insert(table, key, value):
 bucket_idx = hash(key) % table->size
 if found key in table->buckets[bucket_idx]
  replace value
 else:
  add (key, value) into the list
```

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*

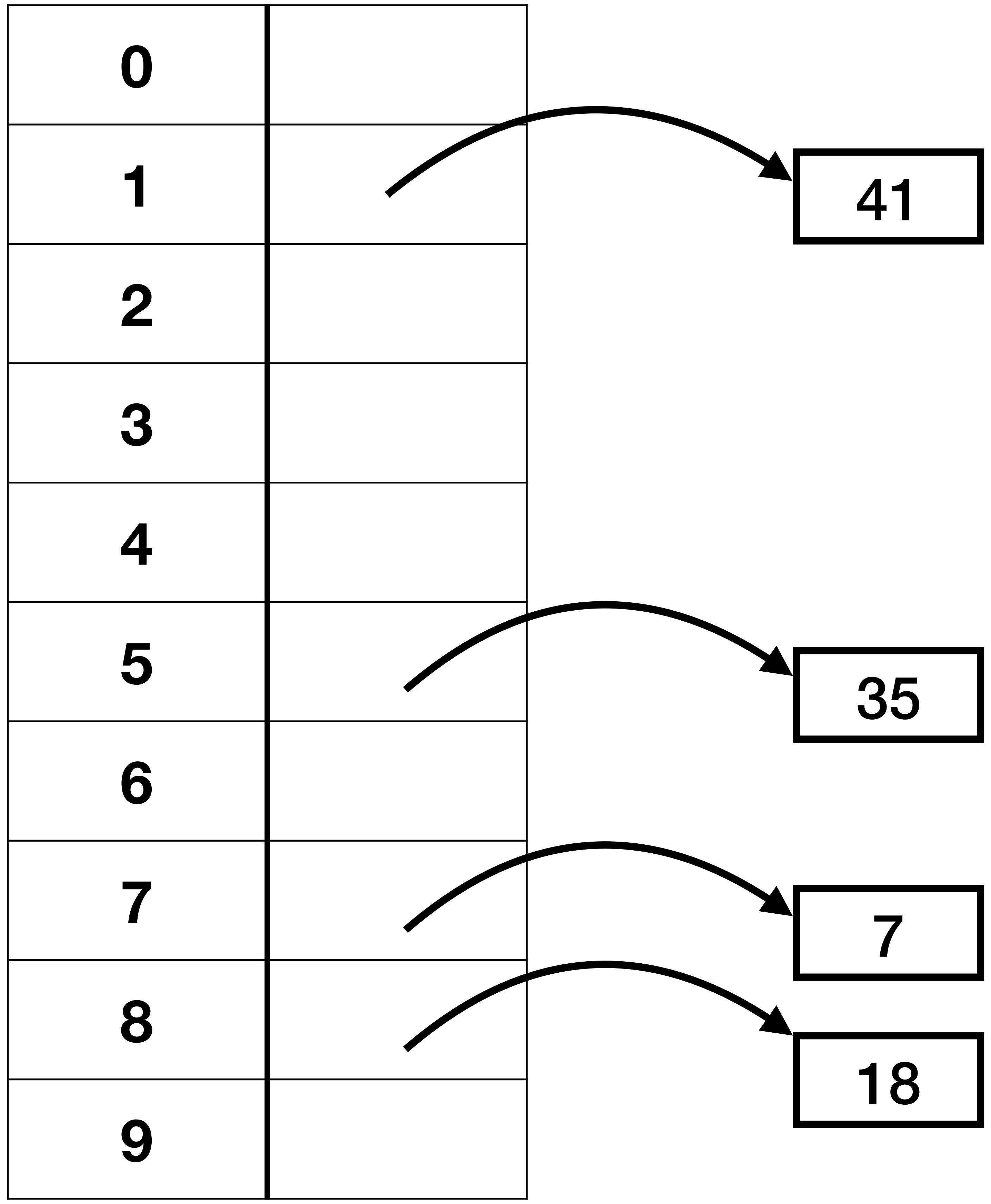



```
struct table {
        int size;
        int length;
        int (*eq)(void *, void *);
        uint64_t (*hash)(void *);
        struct bucket *buckets[];
};

struct bucket {
        void *key;
        void *value;
        struct bucket *next;
};
```

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*




```
struct table {
        int size;
        int length;
        int (*eq)(void *, void *);
        uint64_t (*hash)(void *);
        struct bucket *buckets[];   <<-- what is this?
};

struct bucket {
        void *key;
        void *value;
        struct bucket *next;
};
```

Interlude

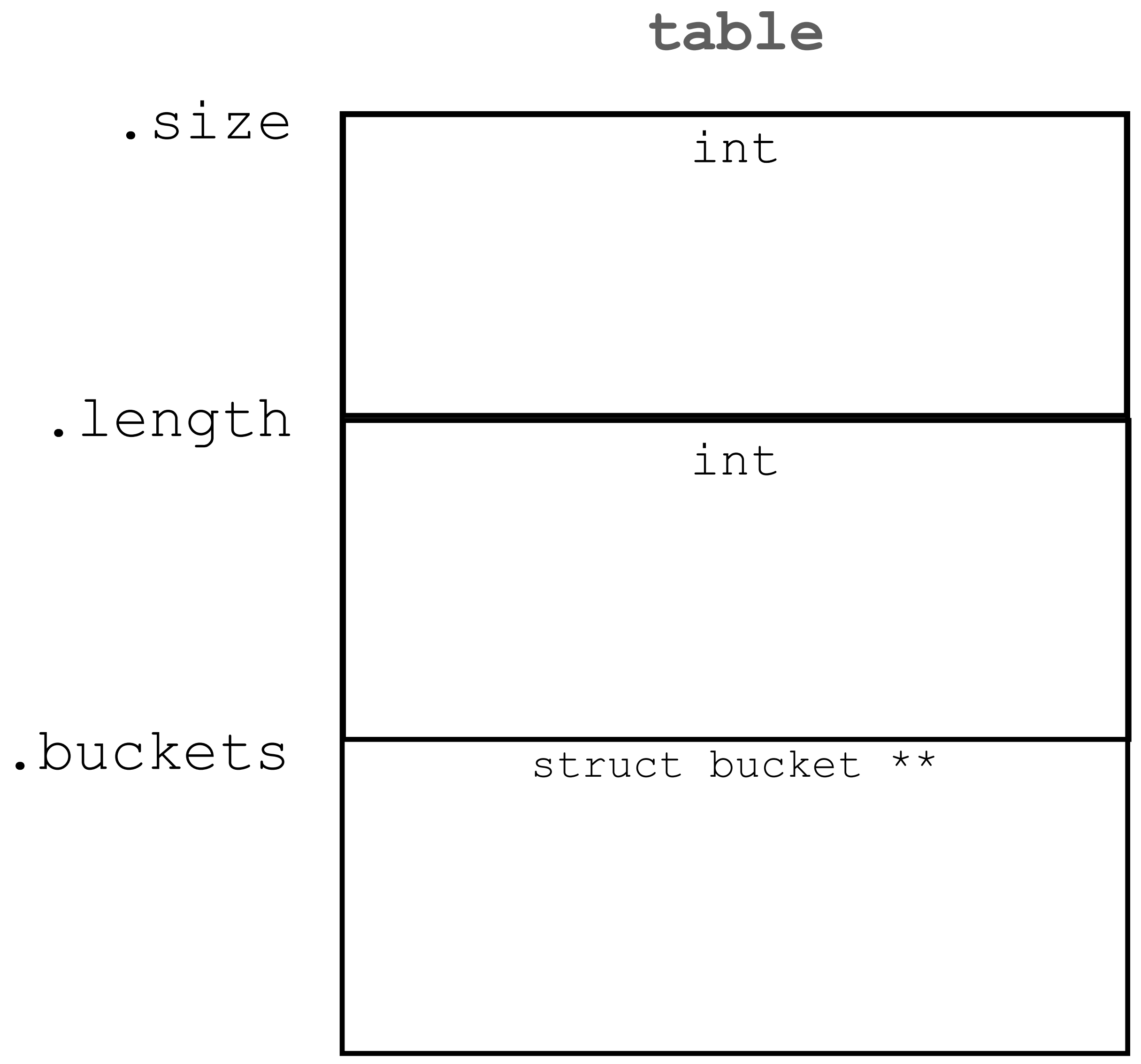
Flexible array member

- The last element of a structure may have an incomplete array type (empty bracket)
- `sizeof` does not include the incomplete field
- Why?

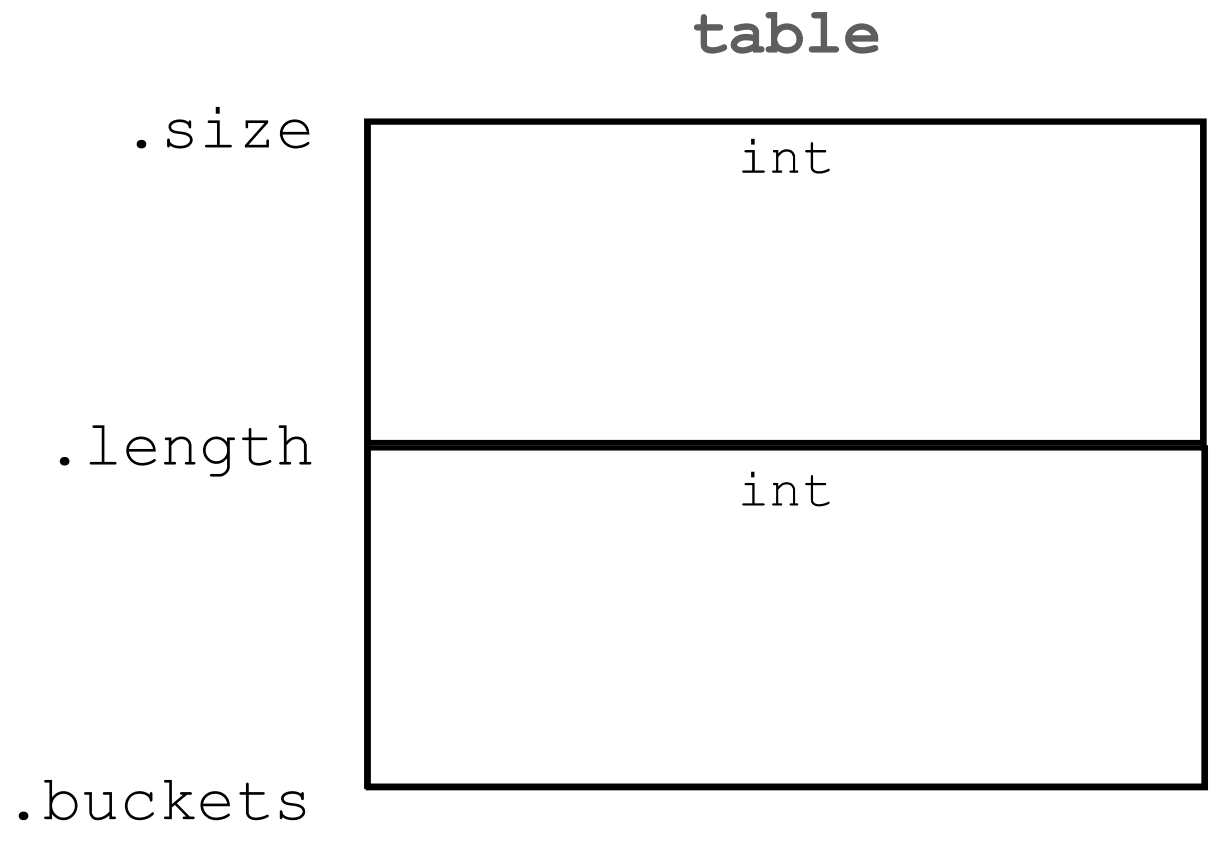
Interlude: Flexible Array Member

Memory Layout

```c
struct table {
        int size;
        int length;
        struct bucket **buckets;
};
```



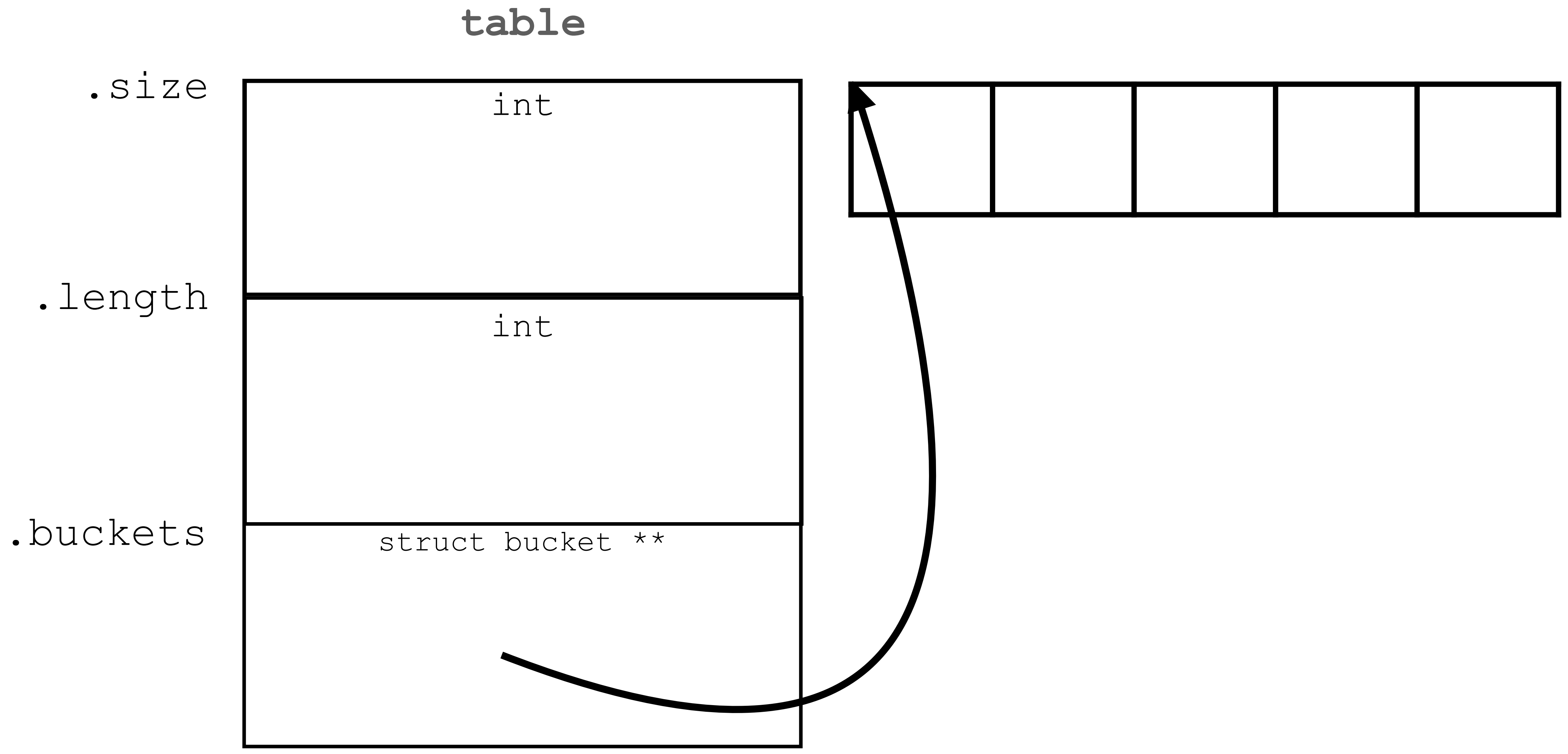

```c
struct table {
        int size;
        int length;
        struct bucket *buckets[];
};
```



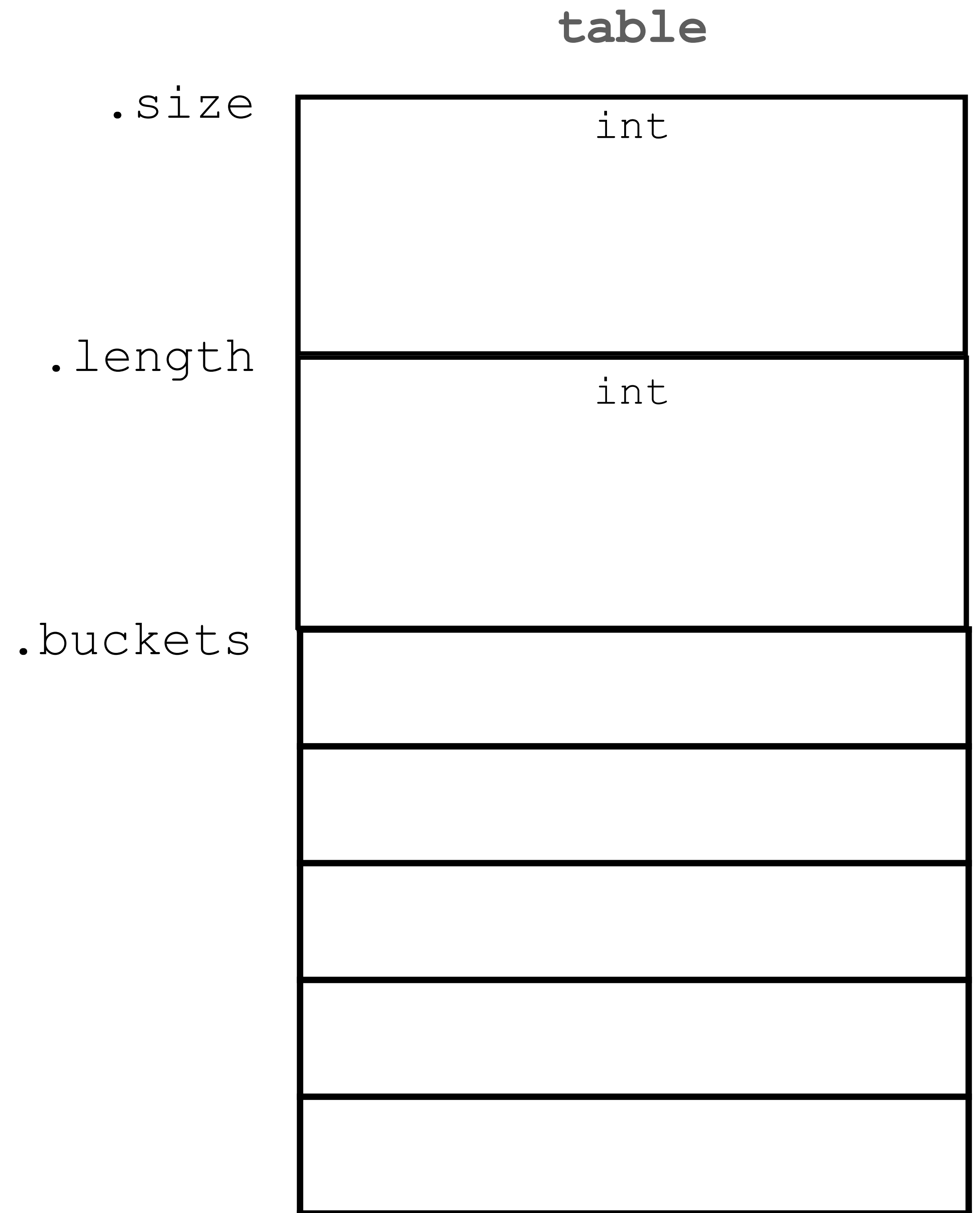
Interlude: Flexible Array Member

Memory Layout

```c
struct table {
        int size;
        int length;
        struct bucket **buckets;
};
```




```c
struct table {
        int size;
        int length;
        struct bucket *buckets[];
};
```



Interlude: Flexible Array Member

Allocation

```c
struct table {
        int size;
        int length;
        struct bucket **buckets;
};

int size = 1024;

struct table *t =
  malloc(sizeof(struct table));

t->buckets =
  malloc(size * sizeof(struct bucket*));
```

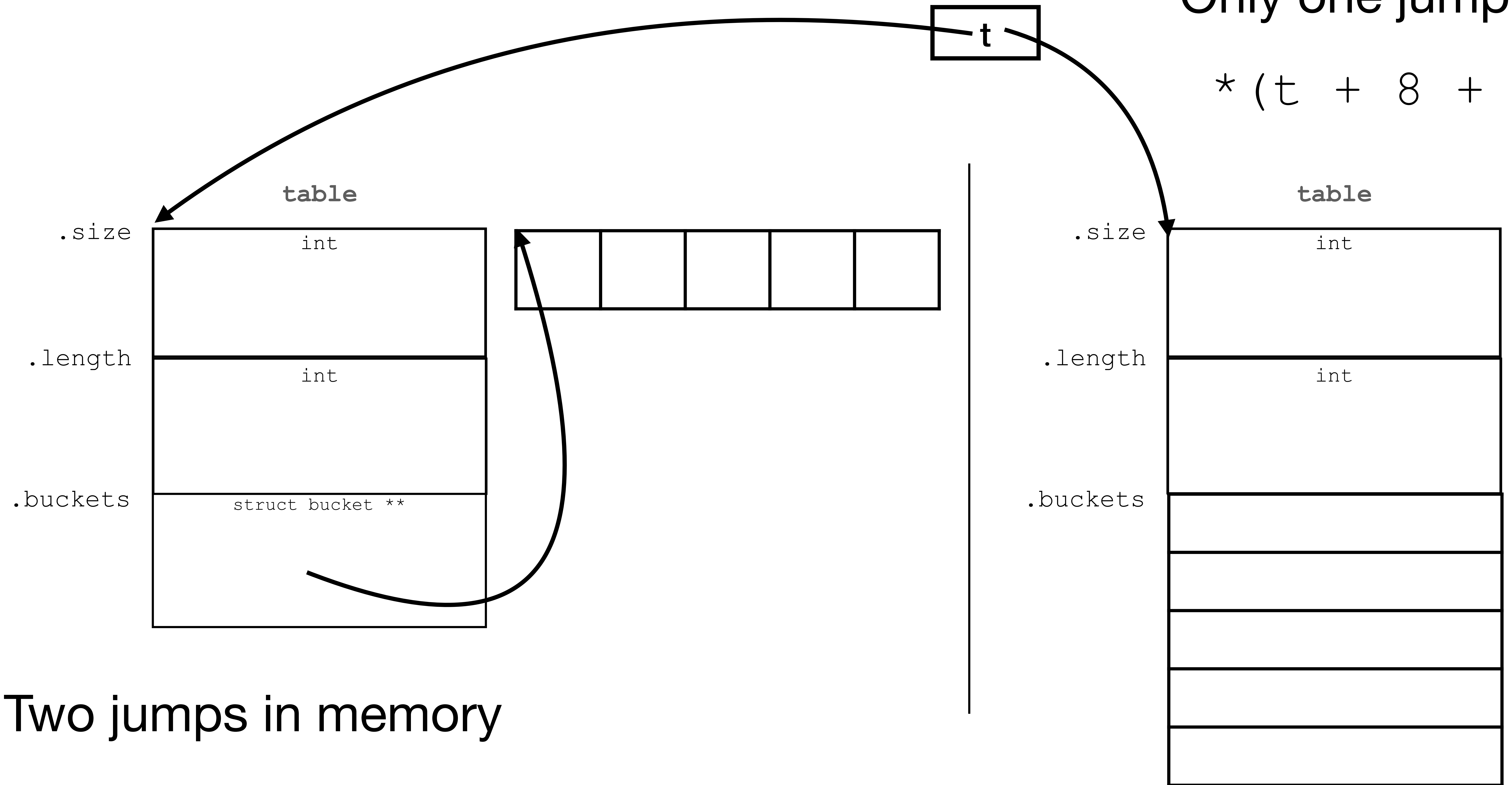
```c
struct table {
        int size;
        int length;
        struct bucket *buckets[];
};

int size = 1024;

struct table *t =
  malloc(sizeof(struct table)
         + size * sizeof(struct bucket*));
```

Interlude: Flexible Array Member

Accessing `t->buckets[3];`

Only one jump!

`*(t + 8 + i * size)`



Two jumps in memory

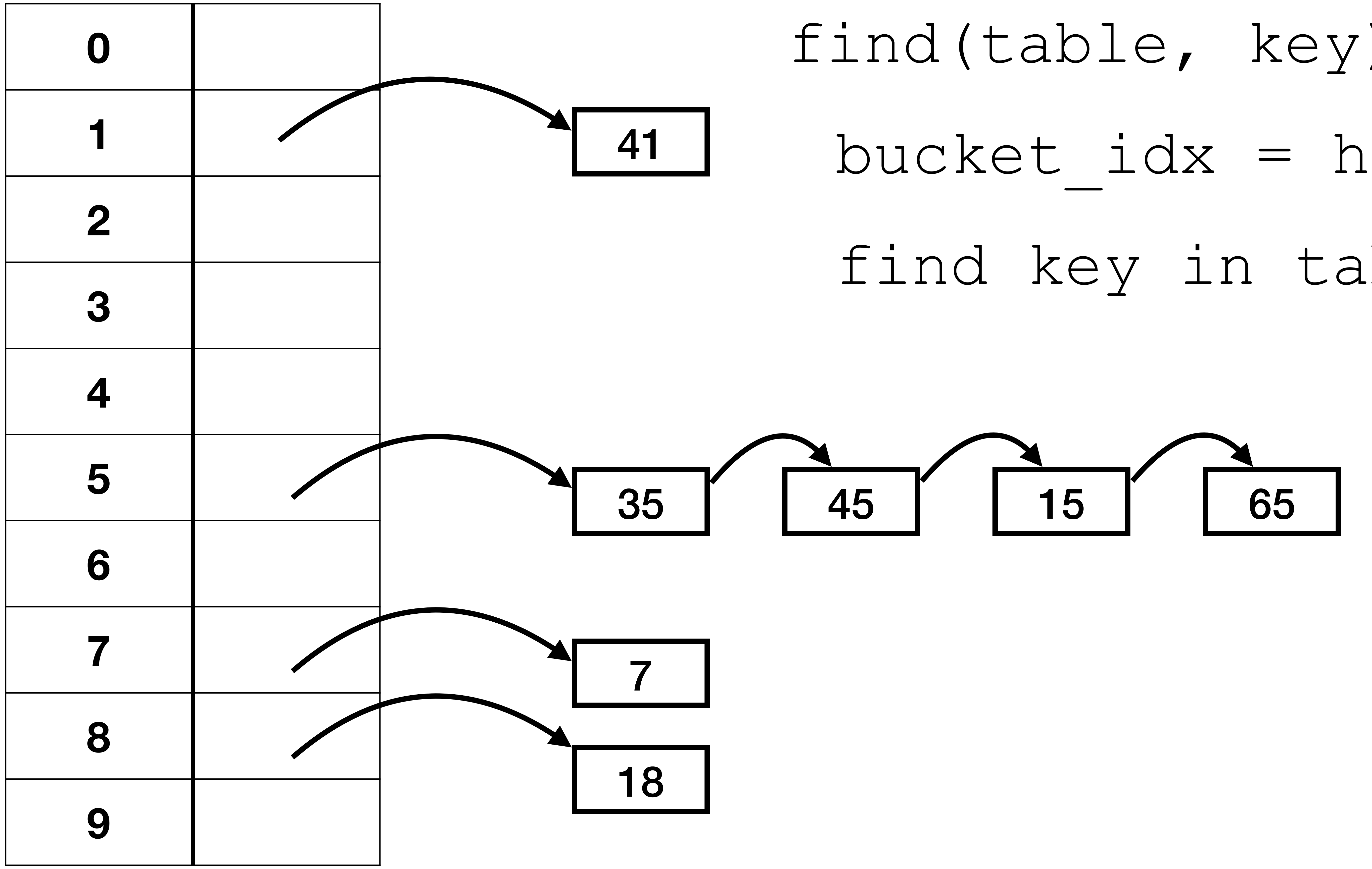
Interlude: Flexible Array Member

- The last element of a structure may have an incomplete array type (empty bracket)
- `sizeof` does not include the incomplete field
- `struct table *ptr = malloc(sizeof(struct table) + extra);`
- Slight performance boost

Chaining

Insert

- Each slot is a *list* of key-value pairs, called a *bucket*




```
find(table, key):
 bucket_idx = hash(key) % table->size
 find key in table->buckets[bucket_idx]:
```

Chaining

Time Complexity

- What is complexity for accessing elements?
  - $O(\text{length of the chain})$
- What is the length of the chain in the worst case?
- $O(n)$
  - This happens for a really bad hash function (e.g. $hash(k) = 1$)
- What if we have a good hash function (that has uniform distribution over a range of integers)?
  - What is the average (expected) length of a chain?
  - $O(\frac{\text{\#elements}}{\text{\#buckets}})$: this ratio is called *load factor*.

Chaining

Time Complexity

- In practice, hash tables are very fast
    - Typically faster than BSTs
- Especially we can keep the load factor $O(1)$
    - Analysis deferred to algorithms

Hash Table

Handling Collision

- Two approaches:
  1. ~~Chaining: put a list in each bucket~~
  2. Probing: use spare space in the array

Probing

- If the bucket is occupied, use the next one.

Probing

- If the bucket is occupied, use the next one.

0	
1	41
2	
3	
4	
5	35
6	
7	7
8	18
9	

Probing

- If the bucket is occupied, use the next one.

0	
1	41
2	
3	
4	
5	35
6	
7	7
8	18
9	

75

Probing

- If the bucket is occupied, use the next one.

0	
1	41
2	
3	
4	
5	35
6	75
7	7
8	18
9	

- Wrap around when reaching the end of array
- The table *must* have some extra space, i.e. load factor has to be ≤ 1
- Many flavors of "next one":
  - *Linear probing*: +1 at a time
  - *Quadratic probing*: * 2 at a time
  - ...

Probing

Linear probing (example)

```c
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

- insert("alice", 400)

Probing

Linear probing (example)

```c
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	
4	
5	("alice", 400)
6	
7	
8	
9	

- insert("bob", 30)

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	
5	("alice", 400)
6	
7	
8	
9	

- insert("carl", 50)

Probing

Linear probing (example)

```c
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	("alice", 400)
6	
7	
8	
9	

- insert("eve", 100)

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	("alice", 400)
6	("eve", 100)
7	("david", 60)
8	
9	

- insert("david", 60)

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	("alice", 400)
6	("eve", 100)
7	("david", 60)
8	
9	

- find("eve")
  - Go to 3 bucket
  - Move down until we find "eve" or until we hit empty bucket
- return 100

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	("alice", 400)
6	("eve", 100)
7	("david", 60)
8	
9	

- find("karl")
  - Go to 4 bucket
  - Move down until we find "karl" or until we hit empty bucket
- No "karl" in table

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	("alice", 400)
6	("eve", 100)
7	("david", 60)
8	
9	

- remove("alice")
    - Go to 5
    - Move down until we find "alice"

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- remove("alice")
  - Go to 5
  - Move down until we find "alice"

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- Find("eve")
  - Go to 3
  - How far do we move down?

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- When we removed "alice" we left a hole
- When searching for "eve" if we stop at the hole, we won't find "eve"
- But if we don't stop at empty spots, we have to search through the entire array if a key doesn't exist

Probing

Linear probing (example)

```
struct bucket {
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- A bucket can be in one of three states:
  - Occupied (key != NULL)
  - Empty, but was always empty
  - Empty, but previously occupied

Probing

Linear probing (example)

true when previously occupied

```
struct bucket {
        bool removed;
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- A bucket can be in one of three states:
  - Occupied (key != NULL)
  - Empty, but was always empty
  - Empty, but previously occupied

Probing

Linear probing (example)

true when previously occupied

```
struct bucket {
        bool removed;
        void *key;
        void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	REMOVED
6	("eve", 100)
7	("david", 60)
8	
9	

- Find("eve")
  - Go to 3
  - Move down until we find "eve", or until we hit an empty, non-removed bucket

Probing

Linear probing (example)

```
struct bucket {
    bool removed;
    void *key;
    void *value;
};
```

true when previously occupied

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- Find("eve")
  - Go to 3
  - Move down until we find "eve", or until we hit an empty, non-removed bucket
- This empty but removed bucket is sometimes called a *tombstone*

Probing

Linear probing

true when
previously occupied

```c
struct bucket {
    bool removed;
    void *key;
    void *value;
};
```

- Let's use `strlen` as our (bad) hash function

0	
1	
2	
3	("bob", 30)
4	("carl", 50)
5	
6	("eve", 100)
7	("david", 60)
8	
9	

- Find/Remove:
  - Move down until first empty bucket
  - If tombstone is encountered, continue searching
- Insert:
  - Move down until first empty bucket
  - If tombstone is encountered, we can reuse that bucket
  - But to avoid inserting duplicate keys, we need to continue searching until an unremoved bucket

Probing

Linear probing

```
struct bucket {
        bool removed;
        void *key;
        void *value;
};
```

- This is why a good hash function spreads out outputs
- If the hash function maps similar inputs to similar outputs, e.g. strlen, we would get clusters in the hash table.
    - Really bad for probing
    - Clusters mean we need to go through more buckets

Probing

Time Complexity

```
struct bucket {
        bool removed;
        void *key;
        void *value;
};
```

- Chaining: worst $O(n)$, average $O(1)$
- What is the worst case complexity when using probing?
    - Insertion: $O(n)$
        - Worst case: all elements are in one cluster, need to go through all to find unfilled bucket
    - Get: $O(\text{table\_size})$
        - Worst case: all empty buckets are tombstones
- On average, the number of probes is at most $1/(1 - \text{load factor})$

Probing

Time Complexity (Appendix)

- Let A_i be the event that the ith probe is occupied.
  - $\Pr[A_1] = n/m$, assuming n elements and m slots
  - $\Pr[A_2] = (n-1)/(m-1)$, since $n-1$ elements and $m-1$ slots are remaining, assuming uniform hashing
  - $\Pr[A_1 \cap A_2 \cap \ldots \cap A_{i-1}] = \frac{n}{m} \cdot \frac{n-1}{m-1} \cdots \frac{n-i+2}{m-i+2} \leq \left(\frac{n}{m}\right)^{i-1}$ = load factor^{i-1}
- $E[\text{\#probes}] = \sum_{i=1}^{\infty} \Pr[A_1 \cap \ldots \cap A_{i-1}]$
- $\leq \sum_{i=1}^{\infty} \text{load factor}^{i-1}$
- $= \sum_{i=0}^{\infty} \text{load factor}^{i}$
- $= \frac{1}{1 - \text{load factor}}$
- E.g. if the table is half full, the average number of probes is 1 / (1 - 0.5) = 2

Load Factor

Notes

- Keep load factor $O(1)$ makes all operations $O(1)$
- Systems typically keep load factor around 0.7 to 0.75
  - This is determined through experimentation
  - Space vs. time trade-off
- What should we do when we hit the maximum load factor?
  - Increase the # of buckets
  - Can we just realloc? I.e. put the same elements in the same buckets after expansion?

Maps

Complexity

	lookup		insert		remove	
	average	worst	average	worst	average	worst
ArrayList	O(n)		O(1)	O(n)	O(1)	
Linked List	O(n)		O(1)		O(1)	
ArrayList (sorted)	O(log n)		O(n)		O(n)	
Linked List (sorted)	O(n)		O(1)		O(1)	
BST	O(log n)	O(n)	O(log n)	O(n)	O(log n)	O(n)
Hash Table	O(1)	O(n)	O(1)	O(n)	O(1)	O(n)

Hash Table

Epilogue

- Hash tables are excellent at insertion, removal, and looking up. What operations are they bad at?
- Operations that involve comparisons:
  - find_min and find_max
  - range look up: give me 10 < key < 20
  - Better to use a heap or BST for these
- Operations that involve ordering, insert "front" and "back"
  - Hash tables have no notion of "order" -- in C++, hash tables are called unordered_map

Hash Table

In one slide

- Array access is $O(1)$.
- Using arbitrary keys as array indices:
  - Hash functions turn any values into an integer. Ideally, this should be uniform.
  - Compress function forces integers into [0, table_size).
- Handling Collision:
  - Chaining: put a list in each bucket
  - Probing: use spare space in the array
- Load factor: the expected number of elements to go through
  - #elements / #buckets
  - Chaining: load factor has no limit; probing: load factor at most 1
  - Adjusting #buckets to keep load factor (0.7 - 0.75) -- time/space trade-off

Data Structures

- Establishing structures on the heap:
  - Indices: contiguous
    - $O(1)$ random access
    - difficult to reorder and reallocate
  - Pointer: scattered
    - sequential access
    - easy to reorder and reallocate

	Indices	**Pointers**
List	Array List	Linked List
Map	Hash Table	BST