CMSC 28100-1 / MATH 28100-1
Introduction to Complexity Theory
Fall 2017 – Homework 4

October 19, 2017

Exercise 1 (HMU 9.3.1)**.** Show that the set of Turing machine codes for TMs that accept all inputs that are palindromes (possibly along with other inputs) is undecidable, that is, show that the language

$$L = \{\langle M \rangle : \forall w \in \Sigma^*, (w = w^R \implies w \in L(M))\}$$

is undecidable, where w^R denotes the reverse of the string w.

Exercise 2 (HMU 9.3.2)**.** The Big Computer Corp. has decided to bolster its sagging market share by manufacturing a high-tech version of the Turing machine, called BWTM, that is equipped with *bells* and *whistles*. The BWTM is basically the same as your ordinary Turing machine, except that each state of the machine is labeled either a "bell-state" or a "whistle-state". Whenever the BWTM enters a new state, it either rings the bell or blows the whistle, depending on which type of state it has just entered. Prove that it is undecidable whether a given BWTM M, on given input w, ever blows the whistle.

Exercise 3 (HMU 9.3.3)**.** Show that the language of codes for TMs that, when started with blank tape, eventually write a 1 somewhere on the tape is undecidable.

Exercise 4 (HMU 9.3.5)**.** Let L be the language consisting of pairs of TM codes plus an integer (M_1, M_2, k) such that $L(M_1) \cap L(M_2)$ contains at least k strings. Show that L is recursively enumerable, but not recursive.