CMSC 23700	**Introduction to Computer Graphics**	**Homework 1**
Winter 2014		**Due January 16**

1. Show that for any two non-zero vectors $\mathbf{u}, \mathbf{v} \in \Re^3$,

$$(\mathbf{u} \cdot \mathbf{v})^2 + |\mathbf{u} \times \mathbf{v}|^2 = \mathbf{u}^2\mathbf{v}^2$$

2. Let $\mathbf{u} = \langle 2, -1, 0 \rangle$ and $\mathbf{v} = \langle 2, 1, -1 \rangle$. Then calculate the following quantities:

   (a) $\mathbf{u} \cdot \mathbf{v}$

   (b) $\mathbf{u} \times \mathbf{v}$

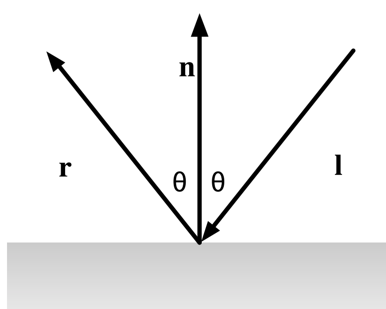
   (c) $\mathbf{v} \times \mathbf{u}$

   (d) $\mathrm{proj}_{\mathbf{u}}\, \mathbf{v}$ (the projection of $\mathbf{v}$ onto $\mathbf{u}$).

3. Prove that for any three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \Re^3$,

$$(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{v} \cdot \mathbf{w})\mathbf{u}$$

4. Consider the following picture, where $\mathbf{n}$, $\mathbf{l}$, and $\mathbf{r}$ are all unit vectors that lie in the same plane. Give an equation for $\mathbf{r}$ in terms of $\mathbf{n}$ and $\mathbf{l}$ (*i.e.*, that does not refer to θ).



5. Consider two unit vectors, $\mathbf{u}$ and $\mathbf{v}$. The *linear interpolation* between these vectors is defined to be

$$\mathrm{lerp}(\mathbf{u}, t, \mathbf{v}) = (1 - t)\mathbf{u} + t\mathbf{v}$$

where $0 \leq t \leq 1$. While this operation works well when the vectors represent positions, it does not work well when the vectors represent directions, since the angle between $\mathbf{u}$ and $\mathrm{lerp}(\mathbf{u}, t, \mathbf{v})$ is not a linear function of t.

Give psuedocode for a function $\mathit{slerp}(\mathbf{u}, t, \mathbf{v})$, where $0 \leq t \leq 1$, that returns a unit vector $\mathbf{w}$, such that the angle between $\mathbf{u}$ and $\mathbf{w}$ is a linear function of t.