

## Last Tutorial

### Additional Questions

### *Not to be Handed In*

1. Show that  $NP$  is closed under the star operation.
2. Let  $HALF - CLIQUE = \{\langle G \rangle \mid G \text{ is undirected graph having a clique of size at least } m/2 \text{ nodes, where } m \text{ is the number of nodes in } G\}$ . Show that  $HALF - CLIQUE$  is  $NP$ -complete.
3. In the following solitaire game, you are given  $m \times m$  board. On each of its  $m^2$  positions lies either a blue stone, a red stone, or nothing at all. You play by removing stones from the board so that each column contains only stones of a single color and each row contains at least one stone. You win if you achieve this objective. Winning may or may not be possible, depending upon the initial configuration. Let  $SOLITAIRE = \{\langle G \rangle \mid G \text{ is a winning game configuration}\}$ . Show that  $SOLITAIRE$  is  $NP$ -complete. *Hint: reduce from 3SAT.*
4. You've seen that  $3SAT$  is  $NP$ -complete. However,  $2SAT$  is (*computationally*) easy. Show that  $2SAT \in P$ .