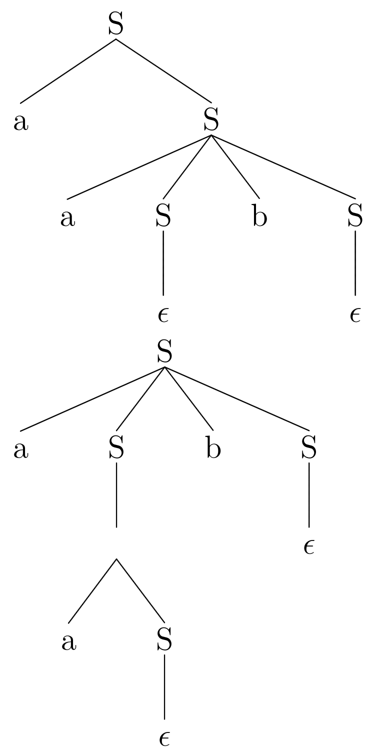
DISCLAIMER: The solutions presented below are incomplete and might be insufficient to get full grade on the homework. They do not model acceptable solutions, but rather present an idea of how a certain problem can be approached. A diligent student should be able to work out complete solutions. Please report any mistakes that you find to the instructor and TA(s).

1. (a)



   (b) Leftmost derivations

      i. $S \implies aS \implies aaSbS \implies aa\epsilon bS \implies aab\epsilon = aab$

      ii. $S \implies aSbS \implies aaSbS \implies aa\epsilon bS \implies aab\epsilon = aab$

   (c) Rightmost derivations

      i. $S \implies aS \implies aaSbS \implies aaSb\epsilon \implies aa\epsilon b = aab$

      ii. $S \implies aSbS \implies aSb\epsilon \implies aaSb \implies aa\epsilon b = aab$

2. Proof by induction on the number of steps n in the derivation.

   Base case: If $n = 0$, then $w = S \implies \epsilon$.

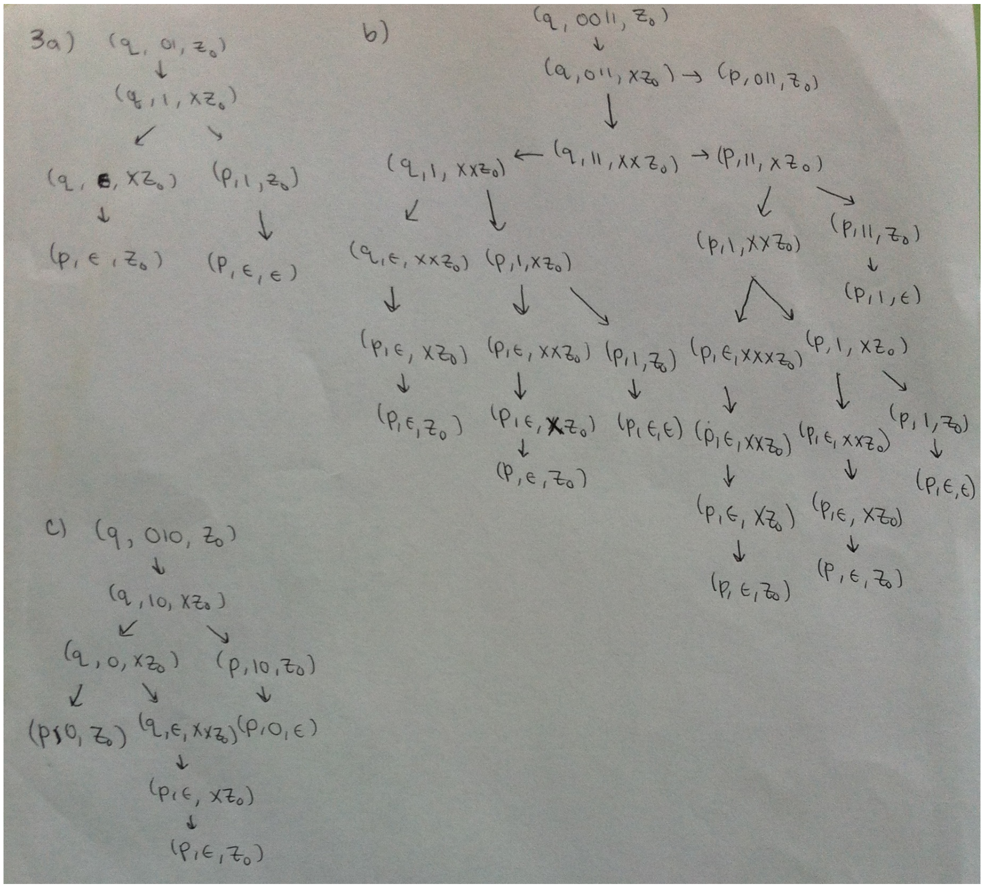
   Induction hypothesis: Assume claim that there any prefix of string w has as many a's as b's holds for n derivation steps. Show that it is true for $n+1$ derivation steps.

   Proof: The $n + 1^{th}$ derivation is of the form

   (a) $S \implies aS \implies * ax = w$. By the induction hypothesis we know any prefix of x has as many a's as b's, and so ax must too.

(b) or $S \implies aSbS \implies * \; ax_1bx_2 = w$. By the induction hypothesis any prefix of x_1 or x_2 has as many a's as b's, so ax_1bx_2 must also have this property.

3. (a)



4. (a) $P = (\{q,p\}, \{0,1\}, \{Z_0, X\}, \delta, q, Z_0, \emptyset)$
$\delta(q, 0, Z_0) = \{(q, XZ_0)\}$
$\delta(q, 1, X) = \{(p, \epsilon)\}$
$\delta(q, 0X) = \{(q, XX)\}$
$\delta(p, 1, X) = \{(p, \epsilon)\}$
$\delta(p, \epsilon, Z_0) = \{(p, \epsilon)\}$

(b) $P = (\{q\}, \{0,1\}, \{Z_0, X\}, \delta, q, Z_0, \emptyset)$
$\delta(q, 0, Z_0) = \{(q, XZ_0), (q, Z_0)\}$
$\delta(q, 1, X) = \{(q, XX), (q, X)\}$
$\delta(q, 0X) = \{(q, \epsilon)\}$
$\delta(q, \epsilon, Z_0) = \{(q, \epsilon)\}$

(c) $P = (\{q\}, \{0,1\}, \{Z_0, X, Y\}, \delta, q, Z_0, \emptyset)$
$\delta(q, 0, Z_0) = \{(q, XZ_0)\}$

$\delta(q, 0, X) = \{(q, XX)\}$
$\delta(q, 0, Y) = \{(q, \epsilon)\}$
$\delta(q, 1, \epsilon) = \{(q, YZ_0)\}$
$\delta(q, 1, X) = \{(q, \epsilon)\}$
$\delta(q, 1, Y) = \{(q, YY)\}$
$\delta(q, \epsilon, Z_0) = \{(q, \epsilon)\}$

5. (a) $P = (\{q, r, p\}, \{a, b, c\}, \{X, Y, Z_0\}, \delta, q, Z_0, \{p\})$
$\delta(q, a, Z_0) = \{(q, XZ_0)\}$
$\delta(q, a, X) = \{(q, XX)\}$
$\delta(q, b, X) = \{(q, YX), (r, \epsilon)\}$
$\delta(r, b, X) = \{(r, \epsilon)\}$
$\delta(r, \epsilon, Z_0) = \{(p, Z_0)\}$
$\delta(q, b, Y) = \{(q, YY)\}$
$\delta(q, c, Y) = \{(q, \epsilon)\}$
$\delta(q, \epsilon, X) = \{(p, X)\}$

(b) $P = (\{p, q\}, \{0, 1\}, \{Z_0, X, Y\}, \delta, q, Z_0, \emptyset)$
$\delta(q, \epsilon, Z_0) = \{(q, \epsilon)\}$
$\delta(q, 0, Z_0) = \{(q, XZ_0)\}$
$\delta(q, 1, Z_0) = \{(q, YYZ_0)\}$
$\delta(q, 1, Y) = \{(q, YYY)\}$
$\delta(q, 1, X) = \{(p, \epsilon)\}$
$\delta(p, \epsilon, X) = \{(q, \epsilon)\}$
$\delta(q, 0, X) = \{(q, XX)\}$
$\delta(p, 0, Y) = \{(q, \epsilon)\}$