Homework 7

due Friday Dec 7 in class

1. Let $L_1, L_2, \ldots, L_k$ be a collection of languages over alphabet Σ such that:

   (a) For all $i \neq j$, $L_i \cap L_j = \emptyset$; i. e., no string is in two of the languages.

   (b) $L_1 \cup L_2 \cup \cdots \cup L_k = \Sigma^*$; i. e., every string is in one of the languages.

   (c) Each of the languages L_i, for $i = 1, 2, \ldots, k$ is recursively enumerable.

   Prove that each of the languages is therefore recursive.

2. We know by Rice's theorem that none of the following problems are decidable. However, are they recursively enumerable, or non-RE?

   (a) Does $L(M)$ contain at least two strings?

   (b) Is $L(M)$ infinite?

   (c) Is $L(M)$ a context-free language?

   (d) Is $L(M) = (L(M))^R$?

3. Let L be the language consisting of pairs of TM codes plus an integer, (M_1, M_2, k) such that $L(M_1) \cap L(M_2)$ contains at least k strings. Show that L is RE, but not recursive.

4. Show that the following questions are decidable:

   (a) The set of codes for TM's M such that, when started with blank tape will eventually write some nonblank symbol on its tape. *Hint:* If M has m states, consider the first m transitions that it makes.

   (b) The set of codes for TM's that never make a move left on any input.

   (c) The set of pairs (M, w) such that TM M, started with input w, never scans any tape cell more than once.

5. Show that the following problems are not recursively enumerable:

   (a) The set of pairs (M, w) such that TM M, started with input w, does not halt.

   (b) The set of pairs (M_1, M_2) such that $L(M_1) \cap L(M_2) = \emptyset$.

   (c) The set of triples (M_1, M_2, M_3) such that $L(M_1) = L(M_2)L(M_3)$; i. e., the language of the first is the concatenation of the languages of the other two TM's.