Homework 5

due Friday Nov 16 in class

1. Use the CFL pumping lemma to show each of these languages not to be context-free:
   (a) $\{a^i b^j c^k \mid i < j < k\}$
   (b) $\{a^n b^n c^i \mid i \leq n\}$
   (c) $\{0^p \mid p \text{ is a prime}\}$

2. Consider the following two languages:

$$L_1 = \{a^n b^{2n} c^m \mid n, m \geq 0\}$$
$$L_2 = \{a^n b^m c^{2m} \mid n, m \geq 0\}$$

   (a) Show that each of these languages is context-free by giving grammars for each.
   (b) Is $L_1 \cap L_2$ context-free? Justify your answer.

3. Give algorithms to decide the following:
   (a) Is $L(G)$ finite, for a given CFG G? Hint: use pumping lemma.
   (b) Does $L(G)$ contain at least 100 strings, for a given CFG G?
   (c) Given CFG G and one of its variables A, is there any sentential form in which A is the first symbol. Note: remember that it is possible for A to appear first in the middle of some sentential form but then for all the symbols to its left to derive ϵ.

4. Design Turing machines for each of the following languages:
   (a) The set of strings with an equal number of 0s and 1s.
   (b) $\{a^n b^n c^n \mid n \geq 1\}$
   (c) $\{ww^R \mid w \in \{0,1\}^*\}$