Homework 4
due Friday Nov 2 in class

1. Consider the grammar
$$S \to aS \mid aSbS \mid \epsilon$$
   This grammar is ambiguous. Show in particular that the string aab has two:

   (a) parse trees,
   (b) leftmost derivations,
   (c) rightmost derivations.

2. Prove that the above grammar from Exercise (1) generates all and only the strings of a's and b's such that every prefix has at least as many a's as b's.

3. Suppose the PDA $P = (\{q, p\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\})$ has the following transition function:

   - $\delta(q, 0, Z_0) = \{(q, XZ_0)\}$
   - $\delta(q, 0, X) = \{(q, XX)\}$
   - $\delta(q, 1, X) = \{(q, X)\}$
   - $\delta(q, \epsilon, X) = \{(p, \epsilon)\}$
   - $\delta(p, \epsilon, X) = \{(p, \epsilon)\}$
   - $\delta(p, 1, X) = \{(p, XX)\}$
   - $\delta(p, 1, Z_0) = \{(p, \epsilon)\}$

   Starting from the initial ID (q, w, Z_0), show all the reachable ID's when the input w is

   (a) 01
   (b) 0011
   (c) 010

4. Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

   (a) $\{0^n 1^n \mid n \geq 1\}$
   (b) The set of all strings of 0's and 1's such that no prefix has more 1's than 0's
   (c) The set of all strings of 0's and 1's with an equal number of 0's and 1's

5. Design a PDA to accept each of the following languages.

   (a) $\{a^i b^j c^k \mid i = j \text{ or } j = k\}$.
   (b) The set of all strings with twice as many 0's as 1's.