Simple Arithmetic Expression (SAE)

1. Inference Rules for *big-step* semantics: $e \Downarrow n$.

$$\frac{}{\texttt{Num}(m) \Downarrow m} \quad (\texttt{Num}_b)$$

$$\frac{e_1 \Downarrow n_1 \qquad e_2 \Downarrow n_2 \qquad p = n_1 + n_2}{\texttt{Plus}(e_1, e_2) \Downarrow p} \quad (\texttt{Plus}_b)$$

$$\frac{e_1 \Downarrow n_1 \qquad e_2 \Downarrow n_2 \qquad p = n_1 \times n_2}{\texttt{Times}(e_1, e_2) \Downarrow p} \quad (\texttt{Times}_b)$$

Alternate, simpler version of the last two rules:

$$\frac{e_1 \Downarrow n_1 \qquad e_2 \Downarrow n_2}{\texttt{Plus}(e_1, e_2) \Downarrow n_1 + n_2} \quad (\texttt{Plus}_b)$$

$$\frac{e_1 \Downarrow n_1 \qquad e_2 \Downarrow n_2}{\texttt{Times}(e_1, e_2) \Downarrow n_1 \times n_2} \quad (\texttt{Times}_b)$$

2. Inference Rules for *small-step* semantics: $e \mapsto e'$.

$$\frac{}{\texttt{Plus}(\texttt{Num}(m), \texttt{Num}(n)) \mapsto \texttt{Num}(m+n)} \quad (1)$$

$$\frac{e_1 \mapsto e_1'}{\texttt{Plus}(e_1, e_2) \mapsto \texttt{Plus}(e_1', e_2)} \quad (2)$$

$$\frac{e_2 \mapsto e_2'}{\texttt{Plus}(n, e_2) \mapsto \texttt{Plus}(n, e_2')} \quad (3)$$

$$\frac{}{\texttt{Times}(\texttt{Num}(m), \texttt{Num}(n)) \mapsto \texttt{Num}(m \times n)} \quad (4)$$

$$\frac{e_1 \mapsto e_1'}{\texttt{Times}(e_1, e_2) \mapsto \texttt{Times}(e_1', e_2)} \quad (5)$$

$$\frac{e_2 \mapsto e_2'}{\texttt{Times}(n, e_2) \mapsto \texttt{Times}(n, e_2')} \quad (6)$$

Note that these encode *left-to-right* evaluation of arguments.

We use these rules to construct derivations of specific transitions.

$$\frac{\dfrac{}{\texttt{Times}(\texttt{Num}(13), \texttt{Num}(4)) \mapsto \texttt{Num}(52)}\,(4)}{\texttt{Plus}(\texttt{Num}(2), \texttt{Times}(\texttt{Num}(13), \texttt{Num}(4))) \mapsto \texttt{Plus}(\texttt{Num}(2), \texttt{Num}(52))}\,(6)$$