

1. [25 points] Rewrite Harper's proof of part 2 of Theorem 11.1 (pp. 74-75). Use a different example of a base case (i.e.  $\text{if}(\text{true}, e_1, e_2) \mapsto_M e_1$ ). For the inductive cases use the left and right search rules for  $+(e_1, e_2)$  (i.e.  $\text{prim}(+, e_1, e_2)$ ).

The base case is fairly simple and similar to Harper's case (a). But for the benefit of the arguments in the inductive cases, be careful to state exactly what is being proved by induction, and how the variables are bound (e.g. where is the stack variable  $k$  quantified). State explicitly what is the Inductive Hypothesis for each inductive case, and don't take any shortcuts in justifying each detail of the argument. For instance, Harper uses the phrase "By the inductive hypothesis, using the enlarged stack, ...". How exactly does the inductive hypothesis support this change to an enlarged stack?

3. [10 points] Use the E-Machine to evaluate the following MinML expression, which computes the factorial of 2. (\* denotes times):

```
apply(fun f(x: int) : int is
      if =(x,0) then 1 else *(x, apply(f, -(x,1))),
      2)
```

You can leave the type components out of the expressions since they do not play a role in evaluation. Feel free to introduce abbreviations for subexpressions that occur repeatedly to make the computation more concise.

3. [20 points] Assuming that  $e$  is a closed, well-typed MinML expression, will the calculations of its value using the C-Machine and using the E-machine use the same number of steps (i.e. transitions)? Explain your reasoning.
4. [25 points] Discuss what issues come up in a proof that the C-machine and the E-machine are equivalent. Try to describe a strategy for such a proof, and discuss how it would differ from the proof that the C-machine and the  $\mapsto_M$  transition system are equivalent (as given by Theorem 11.1 and Corollary 11.3).
5. [20 points] Programming exercise: modify the E-machine implementation of MinML given in the source file E-machine.sml to add the pair expression and fst and snd projection operations that were the subject of Homework 4. Make sure your modified program compiles and test it with the expression from Problem 3 above. Submit your program by emailing it to me as a plain text attachment.