CS 221/321	**Programming Languages**	**Homework 4**
Fall 2009		**Due: Nov 12, 2009**

1. [75 points] We can augment the MinML language by adding pairs (binary Cartesian products). Concretely, this amounts to adding three new expression forms to the abstract syntax, as shown here:

$$e ::= \ldots \mid (e,\, e) \mid \texttt{fst}(e) \mid \texttt{snd}(e)$$

The basic pair expression has the form $(e_1,\, e_2)$, where e_1 and e_2 are arbitrary expressions. Its value is a pair made up of the values of e_1 and e_2. The expression $\texttt{fst}(e)$ projects out the first component of the pair denoted by e, while $\texttt{snd}(e)$ yields the second component. Thus if $v = (2, \texttt{true})$, then $\texttt{fst}(v) = 2$ and $\texttt{snd}(v) = \texttt{true}$. Note that the first and second components of a pair can have different types, and also that those types can be arbitrary; a pair can have primitive values, functions, or pairs as components.

The definition of a value is also extended to include pair values:

$$v ::= \ldots \mid (v,\, v)$$

i.e., a pair of values is a value.

The type expressions are correspondingly extended with a product form:

$$\tau ::= \ldots \mid \tau * \tau$$

As with the function arror operator, the product operator for types is writen using infix notation.

(a) [10 points]. Add new typing rules for the three new expression forms (note that intuitively, a value like $(2, \texttt{true})$ has the product type $\texttt{int} * \texttt{bool}$).

(b) [15 points]. Add new small-step evaluation rules for the transition relation $\mapsto$ to cover the new expression forms. Evaluation of a pair expression should be *left-to-right*, as it is for the arguments of `plus` and `apply`. [Hint: there will be only 6 new rules, two of which will be instructions.]

(c) [10 points]. State the new clauses in the Inversion Theorem (Theorem 9.1, p. 53) and the Canonical Forms Lemma (Lemma 10.2, p. 61) needed to deal with pairs.

(d) [20 points]. Give the new case of the proof of the Progress Theorem relating to pair expressions of the form $(e_1,\, e_2)$.

(e) [20 points]. Give the new cases of the Preservation Theorem relating to expressions of the form $\texttt{fst}(e)$.

In parts (d) and (e), make sure the presentation of the argument is clear, correct, and complete.