CS 15300	**Fundations of Software**	**Homework Solution 5**
Fall 2006		**Due November 3, 2006**

1. [5] Exercise 6.2.1 (p. 367)
Solution:
(a) $((\neg P) \wedge Q) \rightarrow (P \vee R)$
(b) $((P \vee ((\neg Q) \wedge R)) \rightarrow (P \vee R)) \rightarrow (\neg Q)$
(c) $(A \rightarrow (B \vee (((\neg C) \wedge D) \wedge E))) \rightarrow F$

2. [5] Use truth tables to verify the equivalence $A \vee (A \wedge B) \equiv A$.
Solution:

A	B	$A \wedge B$	$A \vee (A \wedge B)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

3. [10] Exercise 6.2.8(b) (p. 367)
Solution:
Let $W_1 = (A \vee B) \wedge (A \rightarrow C) \wedge (B \rightarrow D) \rightarrow (C \vee D)$.

$$\begin{aligned} W_1(A/\text{true}) &= (\text{true} \vee B) \wedge (\text{true} \rightarrow C) \wedge (B \rightarrow D) \rightarrow (C \vee D) \\ &\equiv \text{true} \wedge C \wedge (B \rightarrow D) \rightarrow (C \vee D) \\ &\equiv C \wedge (B \rightarrow D) \rightarrow (C \vee D) \end{aligned}$$

Let $W_2 = C \wedge (B \rightarrow D) \rightarrow (C \vee D)$.

$$\begin{aligned} W_2(B/\text{true}) &= C \wedge (\text{true} \rightarrow D) \rightarrow (C \vee D) \\ &\equiv C \wedge D \rightarrow (C \vee D) \end{aligned}$$

Let $W_3 = C \wedge D \rightarrow (C \vee D)$.

$$\begin{aligned} W_3(C/\text{true}) &= \text{true} \wedge D \rightarrow (\text{true} \vee D) \\ &\equiv D \rightarrow \text{true} \\ &\equiv \text{true} \end{aligned}$$

$$\begin{aligned} W_3(C/\text{false}) &= \text{false} \wedge D \rightarrow (\text{false} \vee D) \\ &\equiv \text{false} \rightarrow D \\ &\equiv \text{true} \end{aligned}$$

Therefore, W_3 is a tautology.

$$\begin{aligned} W_2(B/\text{false}) &= C \wedge (\text{false} \rightarrow D) \rightarrow (C \vee D) \\ &\equiv C \wedge \text{false} \rightarrow (C \vee D) \\ &\equiv C \rightarrow (C \vee D) \end{aligned}$$

Let $W_4 = C \rightarrow (C \vee D)$.

$$\begin{aligned} W_4(C/\text{true}) &= \text{true} \rightarrow (\text{true} \vee D) \\ &\equiv \text{true} \rightarrow \text{true} \\ &\equiv \text{true} \end{aligned}$$

$$\begin{aligned} W_4(C/\text{false}) &= \text{false} \rightarrow (\text{false} \vee D) \\ &\equiv \text{false} \rightarrow D \\ &\equiv \text{true} \end{aligned}$$

Therefore, W_4 is a tautology. Because W_3 and W_4 are a tautologies, W_2 is a tautology.

$$\begin{aligned} W_1(A/\text{false}) &= (\text{false} \vee B) \wedge (\text{false} \rightarrow C) \wedge (B \rightarrow D) \rightarrow (C \vee D) \\ &\equiv B \wedge \text{true} \wedge (B \rightarrow D) \rightarrow (C \vee D) \\ &\equiv B \wedge (B \rightarrow D) \rightarrow (C \vee D) \end{aligned}$$

Let $W_5 = B \wedge (B \rightarrow D) \rightarrow (C \vee D)$.

$$\begin{aligned} W_5(B/\text{true}) &= \text{true} \wedge (\text{true} \rightarrow D) \rightarrow (C \vee D) \\ &\equiv \text{true} \wedge D \rightarrow (C \vee D) \\ &\equiv D \rightarrow (C \vee D) \end{aligned}$$

Because we just proved that $W_4 = C \rightarrow (C \vee D)$ is a tautology, $D \rightarrow (C \vee D)$ is a tautology for the same reasons.

$$\begin{aligned} W_5(B/\text{false}) &= \text{false} \wedge (\text{true} \rightarrow D) \rightarrow (C \vee D) \\ &\equiv \text{false} \rightarrow (C \vee D) \\ &\equiv \text{true} \end{aligned}$$

Therefore, W_5 is a tautology. Because W_2 and W_5 are a tautologies, W_1 is a tautology.

4. [10] Exercise 6.2.9(d) (p. 368)
Solution:

$$\begin{aligned} A \vee B \rightarrow C &\equiv (\neg(A \vee B)) \vee C \\ &\equiv ((\neg A) \wedge (\neg B)) \vee C \\ &\equiv ((\neg A) \vee C)) \wedge ((\neg B) \vee C)) \\ &\equiv (A \rightarrow C) \wedge (B \rightarrow C) \end{aligned}$$

5. [10] Exercise 6.2.10(d) (p. 368)
Solution:

$$
\begin{aligned}
A \rightarrow (B \rightarrow A) &\equiv (\neg A) \vee (B \rightarrow A) \\
&\equiv (\neg A) \vee ((\neg B) \vee A) \\
&\equiv ((\neg A) \vee A) \vee (\neg B) \\
&\equiv \text{true} \vee (\neg B) \\
&\equiv \text{true}
\end{aligned}
$$

6. [10] Exercise 6.2.11(f) (p. 368)
Solution:

$$
\begin{aligned}
(A \vee B) \wedge (C \rightarrow D) &\equiv (A \vee B) \wedge (\neg C \vee D) \\
&\equiv ((A \vee B) \wedge \neg C) \vee ((A \vee B) \wedge D) \\
&\equiv ((A \wedge \neg C) \vee (B \wedge \neg C)) \vee ((A \wedge D) \vee (B \wedge D)) \\
&\equiv (A \wedge \neg C) \vee (B \wedge \neg C) \vee (A \wedge D) \vee (B \wedge D)
\end{aligned}
$$

7. [10] Exercise 6.2.12(f) (p. 368)
Solution:

$$
\begin{aligned}
(A \wedge B) \vee E \vee F &\equiv (A \wedge B) \vee (E \vee F) \\
&\equiv (A \vee (E \vee F)) \wedge (B \vee (E \vee F)) \\
&\equiv (A \vee E \vee F) \wedge (B \vee E \vee F)
\end{aligned}
$$

8. [40] Write (and test and debug) a program in your favorite language that will evaluate the truth of a WFF given a truth assignment (or interpretation) for its propositional variables. You will need to do the following:

(a) Define a data structure representing WFFs, including propositional variables. You might, for instance, represent propositional variables as strings, or possibly as numbers (integers, say).

(b) Define a data structure or other representation for truth assignments. You could, for instance, use an *association list* data structure consisting of a list or sequence of ordered pairs, where each ordered pair would consist of a propositional variable and its assigned truth value. E.g. the assignment

$$\{\mathsf{P} \mapsto \mathsf{T}, \mathsf{Q} \mapsto \mathsf{F}, \mathsf{R} \mapsto \mathsf{T}\}$$

would be represented by the association list (using ML list syntax):

$$[(\mathsf{P}, \mathsf{T}), (\mathsf{Q}, \mathsf{F}), (\mathsf{R}, \mathsf{T})]$$

(c) Define an evaluator that takes as parameters a WFF and a truth assignment, and returns true or false as appropriate.