CS 153	**Foundations of Software**	**Homework 4**
Fall 2006		**Due: Oct 27, 2006**

1. [5] Suppose $\langle B, \leq \rangle$ is a poset and $f : A \to B$ is a total function. Give two ways of a defining partial order on the domain A such that f is monotonic, and say under what circumstances (if any) these two definitions will coincide. [What I am looking for is the "natural" way of inducing an ordering on A such that f is monotonic, and a "trivial" way of defining an order such that f is monotonic.]

2. Exercise 4.4.2 (b) (p. 267) [5]

3. Exercise 4.4.8 (p. 268) [10]

4. Exercise 4.4.19 (b) (p. 270) [10]

4. [20] (Generalized product). Let I be a nonempty set, which we will call an *index* set. A family of sets indexed by I, which we write as $\{X_i \mid i \in I\}$ is just a function $F : I \to \mathscr{P}(U)$, where the set U is some universe such that each $X_i = F(i) \subseteq U$. For example, if $\langle A, \leq \rangle$ is a poset, we can define the family of initial segments of A by letting $I = A$ and $X_i = s(i)$, where $s(i) = \{x \in A \mid x < i\}$. Note that $X_i = \emptyset$ if i is minimal in A. [What can we use as the universe U in this example?]

Now assume that the elements in a family $\{X_i \mid i \in I\}$ are all nonempty, i.e. $X_i \neq \emptyset$ for each $i \in I$. The generalized product of this family is the set

$$\Pi_{i \in I} X_i = \{f : I \to \cup_{i \in I} X_i \mid \forall i \in I. f(i) \in X_i\}$$

Note that the function space $A \to B$ of total functions from A to B is the same as the generalized product $\Pi_{a \in A} X_a$ where $X_a = B$ for all $a \in A$.

(a). Let $I = \{0, 1\}$ and define the family $\{X_i \mid i \in I\}$ by $X_0 = A$ and $X_1 = B$. Define a function $g : A \times B \to \Pi_{i \in I} X_i$ so that g is a bijection and $fst(p) = (g(p))(0)$ and $snd(p) = (g(p))(1)$ for any $p \in A \times B$. [Here *fst* and *snd* are the first and second projections on ordered pairs, such that $fst(a, b) = a$ and $snd(a, b) = b$.]

(b). Now assume that each X_i is a (nonempty) well-founded poset with ordering $\leq_i$, and define the pointwise ordering of $\Pi_{i \in I} X_i$ by

$$f \leq_p g \iff \forall i \in I. f(i) \leq_i g(i)$$

Give two examples of such pointwise ordered families where the ordering is well-founded and non-well-founded, respectively.