CS 153	Foundations of Software	Homework 3
Fall 2006		Due: October 20, 2006

1. [5] Express the properties of a partial order relation R being antisymmetric and total in terms R and its inverse R^{-1}.

2. Exercise 4.3.14 (b,d,f,h) (p. 253) [20 points]

3. [10] Show that the composition of two monotonic functions between posets is a monotonic function.

4. [5] Show that for any function $f : A \to B$, the associated image function $f : \mathcal{P}(A) \to \mathcal{P}(B)$ and the inverse image function $f^{-1} : \mathcal{P}(B) \to \mathcal{P}(A)$ are monotonic relative the the subset ordering on $\mathcal{P}(A)$ and $\mathcal{P}(B)$.

5. [15] Consider the set $\mathcal{R}$ of binary relations over a set A:

$$\mathcal{R} = \mathcal{P}(A \times A)$$

and let these relations be ordered by subset, so we are considering the poset $\langle \mathcal{R}, \subseteq \rangle$. The closure operations t, s, and r defined in Section 4.1 of the text are unary operations on $\mathcal{R}$, e.g. $t : \mathcal{R} \to \mathcal{R}$. Show that all three of these operations are monotonic.

6. [15] Consider the set of partitions of a set A, ordered by the refinement order: $P_1 \leq P_2$ iff $\forall X \in P_1. \exists Y \in P2. X \subseteq Y$. Show that every set of partitions of A has a lub and a glb. [**Hint**: consider the meaning of the lub and glb in terms of the equivalence relations associated with the partitions.]

7. [10] Given a poset $\langle A, \leq \rangle$, a subset $C \subseteq A$ is a *co-chain* if no two elements in C are comparable (i.e. related by $\leq$.
(a) For the set $\mathcal{P}(\{a, b, c, d\})$ ordered by subset, give the largest maximal co-chain (a maximal co-chain is a co-chain that is not a proper subset of a larger co-chain).
(b) Give an example of a poset with an infinite co-chain.

8. [10] Show that if A is an infinite set, then $\mathcal{P}(A)$ is not well-founded.

9. [10] A preorder is a relation $R \subseteq A \times A$ such that R is reflexive and transitive. It should be clear that given any binary relation Q on A, the reflexive, transitive closure $tr(Q)$ is a preorder. Show that given any preorder R on A, there is an equivalence relation $\sim_R$ on A such that $R/\!\sim_R$ is a partial order on $A/\!\sim_R$, where $R/\!\sim_R = \{([a], [b]) \mid R(a, b)\}$.