

Basic Math concepts, terminology, and notations

1. sets, relations, functions, orders

(ref: Naive Set Theory by Halmos)

sets

set notation

listing elements: $\{1,3,2,5\}$, $\{\}$

comprehension: $\{x \text{ in } A \mid P(x)\}$

Russell's Paradox

set membership

set equality

subset relation, proper subset

singleton set

disjoint sets

operations:

union, intersection, difference, symmetric difference, complement, powerset,

cartesian product (assumes notion of pairs or tuples)

Venn diagrams

characteristic function of a (sub)set

size of a set, cardinality

finite sets, infinite sets

countable sets (functions)

families of sets (functions)

unions, intersections of families

some useful sets:

empty set (\emptyset)

$1 = \{\emptyset\}$ (canonical one element set)

$2 = \text{bool}$

Nat (aka ω)

bags or multisets

relations

a subset of a cartesian product

operations:

inverse

composition (self composition, powers)

image, inverse image

properties (of binary relations)

reflexive, irreflexive

transitive

symmetric

anti-symmetric

closures

reflexive, transitive closures

functions

a special form of binary relation between two sets

functions are assumed partial by default

application of a function

graph of a function

domain, co-domain, range

partial, total functions

injective (1-1), surjective (onto), bijective (1 to 1, onto) functions

inverse function f^{-1} (same as inverse relation)

composition of functions

image, inverse image (inherited from relations)

closure of a set under a function

fixed point of a function

orders

a special form of binary relation on a set

preorder

partial and total orders

linear orders

- well-founded order
- incomparable elements
- minimal and maximal elements
- least and greatest elements
- supremum (sup, lub), infimum (inf, glb) of a set of elements
- chains, co-chains
 - limits of chains (complete partial orders (CPOs))
- pointwise extension of an order to functions
- ordinal numbers
- sorting sequences
- monotone and antimonotone functions on ordered sets

- equivalence relations
 - reflexive, transitive, antisymmetric
- equivalence class
- partitions
- congruence relations

- lattices
 - meet, join
 - induced ordering (every lattice is an ordered set)
 - (meet = binary inf, join = binary sup)
 - eg: powerset as lattice

- boolean algebras
 - generalized and, or, not
 - special form of lattice

- simple structures
 - tuples (elements of a cartesian product)
 - sequences
 - lists (nil, cons, hd, tl, ...) [sequential access]
 - vectors (arrays - update) [random access]
 - strings (vector of characters)
 - heterogeneous and homogeneous
 - sums (tagged disjoint unions)
 - trees
 - graphs

2. Basic Logic

- propositional calculus
 - logical connectives
 - and, or, not, implies, equiv
 - propositional variables
 - models, truth assignments
 - material implication
 - contrapositive
 - truth tables
 - tautologies
 - proofs
 - axioms
 - inference rules

- predicate calculus
 - propositional connectives + terms, relations, variables, quantifiers
 - terms
 - relations
 - variables
 - atomic formulae (literals)
 - quantifiers - universal (all) and existential (exists)
 - axioms and inference rules
 - law of the excluded middle
 - proof by contradiction

[resolution]

Other logics

- intuitionistic logic
- modal and temporal logic
- higher-order logic
- type theory

3. Computing models

lambda calculus: computing with functions

- syntax (concrete and abstract)
- pure and applied
- beta-reduction (function application)
 - beta-redexes
- alpha-conversion (meaninglessness of names of bound variables)
- reduction sequences
- Church-Rosser property
- reduction strategies
 - normal order
 - call-by-name
 - call-by-value
- recursion and the Y-combinator
- simply typed lambda calculus
 - types
 - typing rules
 - well-typed expressions

first-order functional language (informal)

- function equations
- term rewriting systems

finite state machines

- regular expressions

computability

- Church's thesis
- undecidability of the halting problem

4. Fixed point theory and induction

the Tarski-Knaster fixed point theorem

semantics of inductive definitions

- inductive definitions as fixed points

anatomy of inductive proofs

- case analysis
- induction hypotheses