Algorithms – CS-27200/37000 Homework – February 16, 2005

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TA SCHEDULE: TA sessions are held in Ryerson-255, Tuesday and Thursday 5–6pm, Saturday 11am–noon, and (this is new) **Wednesday after class** 12:30–1:20 or 1:30–2:20 depending on demand. Indicate your interest in the Wednesday session to the instructor immediately after class. (The Wednesday evening sessions are discontinued.)

ADVICE. Take advantage of the TA sessions.

Check the class website, `http://www.classes.cs.uchicago.edu/current/27200-1`.

HOMEWORK. Please **print your name on each sheet.** Print "U" next to your name if you seek 27200 credit and "G" if you seek 37000 credit. Undergraduates receive the stated number of points as *bonus points* for "G only" problems. – Please try to make your solutions readable. Unless expressly stated otherwise, all solutions are due at the **beginning of the next class.**

DATES TO REMEMBER: Mon Feb 21: Quiz 2, Mon Mar 7: Midterm 2, Fri Mar 11: Last class. ATTENDANCE REQUIRED. Fri Mar 18, 10:30–12:30: Final Exam

14.1 (Note: in the class handout, this problem was numbered 13.1 by mistake. Please use 14.1.) A linear inequality in the variables x, y, z is an inequality of the form $3x - 5y + z \le 6$. (We may replace the coefficients $3, -5, 1, 6$ by arbitrary real numbers, and the number of variables may also be arbitrary. We shall assume, however, that *all coefficients are integers.*) The *linear programming* (LP) problem takes a system of linear inequalities as input (m inequalities in n variables) and asks its *feasibility* (does there exist a solution, satisfying all the given inequalities at the same time?). The *integer linear programming* (ILP) problem asks the existence of a solution in *integers* (each variable must take an integral value). A $(0,1)$-ILP asks the existence of a solution where each variable takes the value 0 or 1.

LP is one of the most widely applied algorithmic problems. **Fact.** LP is solvable in polynomial time (Khachiyan, 1979, Karmarkar, 1983).

(i) (3 points) Give an example of an LP (with integer coefficients) which is feasible (has solution(s)) but which is not feasible as an ILP. Use as few variables as possible.

(ii) (6 points) Same as item (i) but all coefficients in the LP must be 0, 1, or -1, including the numbers on the right hand side. (A correct solution to this question also earns you the 3 points for part (i) so no separate solution to (i) is required to get all the 6 points.)

(iii) (5 points) Prove that (0,1)-ILP belongs to NP. State the (polynomial-time verifiable) witness (of the yes-answers). Indicate why it is verifiable in polynomial time (length of input to verification algorithm, estimated running time of verification).

(iv) (G only, 6 points) Is it evident that ILP belongs to NP? Argue your answer. Be as specific about the potentially difficult technical detail as you can.