Algorithms – CS-27200/37000 Homework – February 4, 2005

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TA SCHEDULE: TA sessions are held in Ryerson-255, Tuesday and Thursday 5–6pm, Saturday 11am–noon, and (this is new) **Wednesday after class** 12:30–1:20 or 1:30–2:20 depending on demand. Indicate your interest in the Wednesday session to the instructor immediately after class. (The Wednesday evening sessions are discontinued.)

ADVICE. Take advantage of the TA sessions.

Check the class website, http://www.classes.cs.uchicago.edu/current/27200-1.

HOMEWORK. Please **print your name on each sheet.** Print "U" next to your name if you seek 27200 credit and "G" if you seek 37000 credit. Undergraduates receive the stated number of points as *bonus points* for "G only" problems. – Please try to make your solutions readable. Unless expressly stated otherwise, all solutions are due at the **beginning of the next class.**

IMPORTANT. Monday February 7: University undergraduate break, no undergraduate class. But: yes, there will be a grad-only class. Ambitious undergraduates are most welcome to attend – the material will be delightful. ALL HOMEWORK ASSIGNED TODAY IS DUE Wednesday, February 9, unless expressly stated otherwise. Homework problem 8.4 was mistakenly assigned due February 7; it is due February 9.

11.1 (5 points) The Floyd-Warshall algorithm solves the *all-pairs min-cost path* problem on a weighted digraph with no negative-weight cycles (but negative-weight edges are permitted). Study the problem and the algorithm (text, p. 629). Switch lines 3 and 4 in the pseudocode of the Floyd–Warshall algorithm given at the bottom of p. 630 in the text (i. e., switch the order of the first two **for**-loops). Show that this modified code is incorrect. You need to build a counterexample; make it as small as possible. Show the result given by the modified code and compare it with the correct result given by Floyd–Warshall.

11.2 (4 points) Let x, y be integers; let $a = xy$ and $b = (x-1)(y-1)$. Given a, b, compute x and y in polynomial time (polynomial number of bit-operations). Estimate the time required in terms of n where both a and b are n-bit integers. Do not use fancy multiplication such as Karatsuba-Ofman, just the schoolbook $O(n^2)$ algorithm.

11.3 (G only, 10 points. Due Friday, February 11.) Let G be a connected undirected graph with weighted edges. The max-weight of a spanning tree is the weight of its heaviest edge. A min-max spanning tree is a spanning tree of smallest possible max-weight. Find a min-max spanning tree in linear time. Prove your timing.