Algorithms – CS-27200/37000 Homework – January 28, 2005

Instructor: László Babai Ry-164 e-mail: laci@cs.uchicago.edu

NEW! The class now has a website,

http://www.classes.cs.uchicago.edu/current/27200-1

Please check it before you do the homework.

ADVICE. Take advantage of the TA sessions.

TA SCHEDULE: TA sessions are held in Ryerson-255, Tuesday and Thursday 5–6pm, Saturday 11am–noon, and (this is new) **Wednesday after class** 12:30–1:20 or 1:30–2:20 depending on demand. Indicate your interest in the Wednesday session to the instructor immediately after class. (The Wednesday evening sessions are discontinued.)

IMPORTANT. If you have not done so yet, please send e-mail to the instructor with your name, major, year, type of credit sought (letter grade, P/F, etc.), list of proof-oriented math courses previously taken; include whether or not you took CMSC-27100 (Discrete Math). In the subject write 27200 info or 37000 info, as appropriate.

HOMEWORK. Please **print your name on each sheet.** Print "U" next to your name if you seek 27200 credit and "G" if you seek 37000 credit. Undergraduates receive the stated number of points as *bonus points* for "G only" problems. – Please try to make your solutions readable. Unless expressly stated otherwise, all solutions are due at the **beginning of the next class.**

Homework is collected in three separate piles (U, G, "G only"). Please put your solutions to "G only" problems on that pile, and your solutions to other problems on the "U" or "G" pile according to the credit you seek.

GRAD READING: DFS, especially classification of edges, "White Path Theorem," application to toplogical sort. Strassen's algorithm for matrix multiplication.

REMEMBER: **Midterm** Wednesday, Feb 2.

10.1 (U,G) (8 points) We are given an array of real numbers $x[1], \ldots, x[n]$. The sum of the interval $[i, j]$ is the quantity $S[i, j] := \sum_{k=i}^{j} x[k]$. Find the maximum interval sum $S_{\max}$. Find this value in *linear* time (i. e., the number of operations should be $O(n)$). Describe your solution in pseudocode.

(The solution should be very simple, no more than a few lines. **Elegance counts.** *Hint*: dynamic programming.)

Note: you are not required to output the interval with the maximum sum, just the value of the maximum sum. Observe the following convention:

Convention. If $j < i$, we say that the interval $[i, j]$ is *empty*; the sum of the empty interval is zero. Empty intervals are admitted in the problem. Therefore $S_{\max} \geq 0$ even if all the $x[i]$ are negative.

10.2 (U,G) (Due Friday, February 4) We say that a list of real numbers is sorted with $\leq k$ errors if one can remove k items such that the remaining list is sorted. For instance, the list 1, 3, 7, 4, 8 is sorted with one error (remove 7, or remove 4); the list 1, 6, 4, 5, 8, 7 is sorted with two errors (remove 6 and 8, or remove 6 and 7).

(a) (7 points) Given an array $A[1 \ldots n]$ of reals, sorted with one error, sort the array in $O(n)$ steps (comparisons and bookkeeping (link updates)). Describe your algorithm in English as well as in PSEUDOCODE. Don't forget to DEFINE your variables.

(b) (G only, 7 points) Given an array $A[1 \ldots n]$ of reals, sorted with $\leq k$ errors, sort the array in $O(n + k \log k)$ steps. Describe your algorithm in English as well as in PSEUDOCODE.

(c) (G only, 7 points) The *depth* of a sorting network is the number of rounds of parallel comparisons made by the network. Professor Mixup constructed a sorting network which he claims correctly sorts any input which is sorted with one error. Professor Mixup's sorter has depth $O(1)$. Prove that Professor Mixup is wrong; in fact, any sorting network that solves this problem must have depth $\Omega(\log n)$.