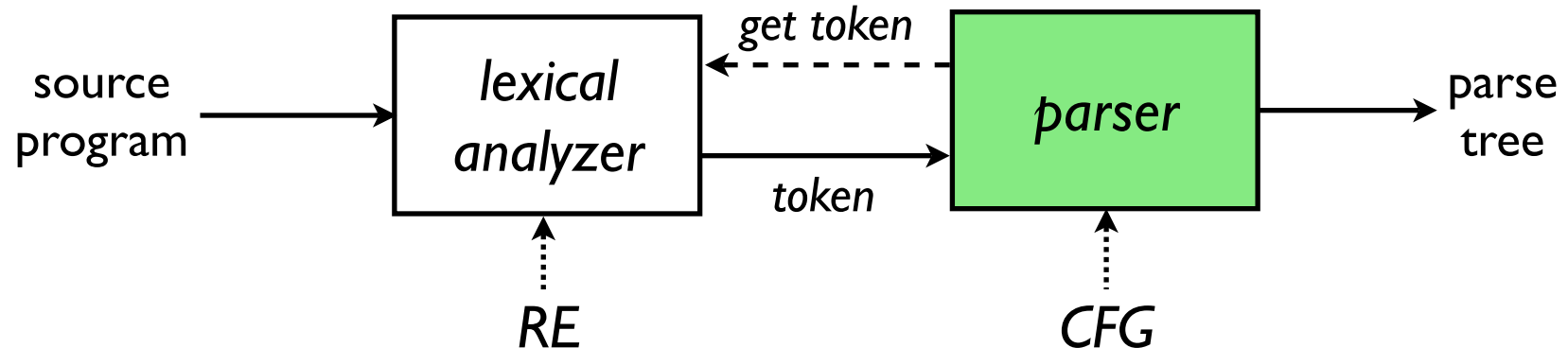


# Lecture 3

## Parsing

# Syntax Analysis

- Transform a sequence of **tokens** into a **parse tree**:



- Syntactic structure is specified using **context-free grammars**
- A parse tree is a representation of the hierarchical structure of a phrase in the language.
- Secondary tasks: syntax error detection and recovery

# Syntax Analysis

function f(a:int,b:string) = g(1+a)

## Tokens

**FUNCTION**

**ID(f)**

**LPAREN**

**ID(a)**

**COLON**

**ID(int)**

**COMMA**

**ID(b)**

**COLON**

**ID(string)**

**RPAREN**

**EQ**

**ID(g)**

**LPAREN**

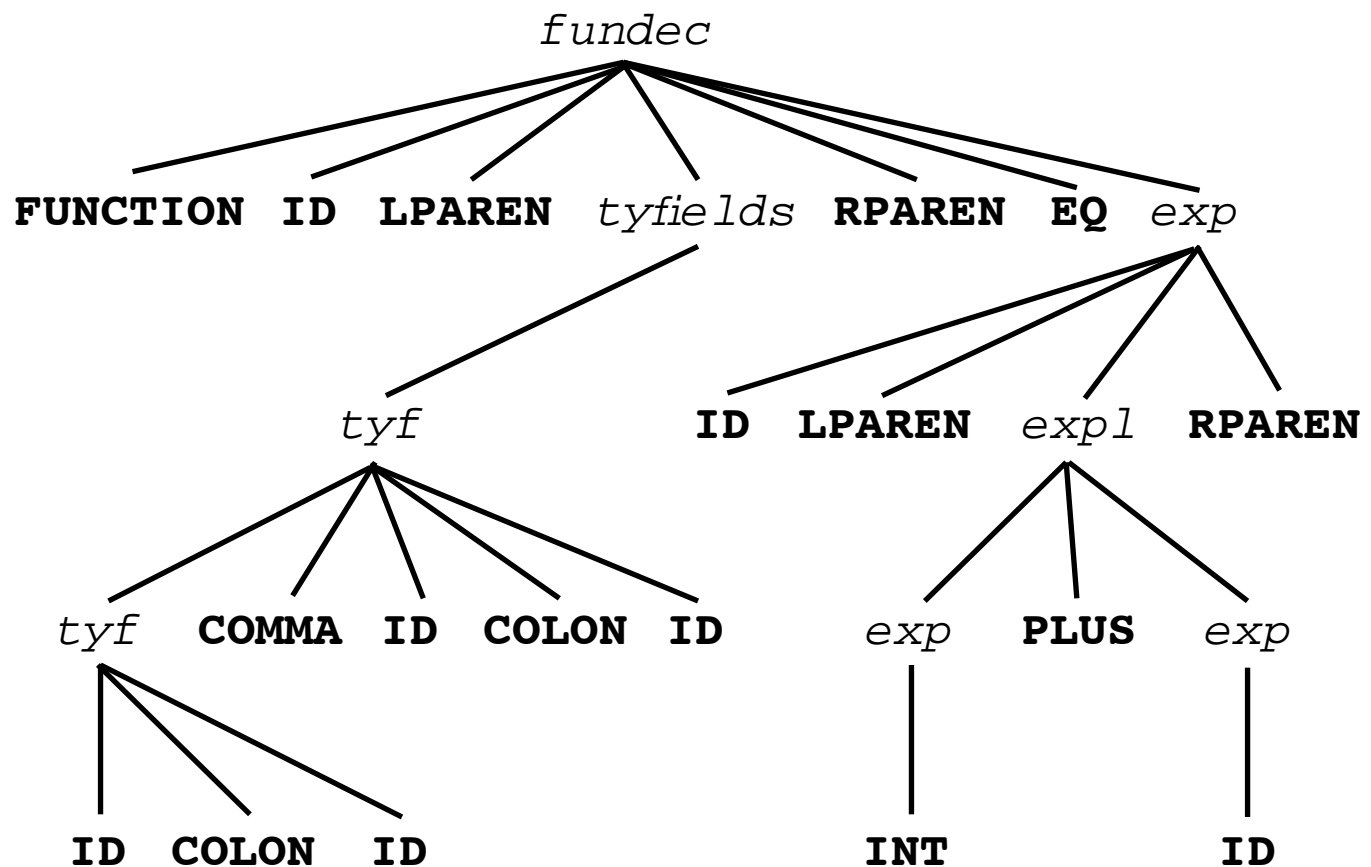
**INT(1)**

**PLUS**

**ID(a)**

**RPAREN**

## Parse Tree



# Main Parsing Problems

- *How to specify the syntactic structure of a programming language?*  
use *Context-Free Grammars (CFG)*
- *How to parse: given CFG and token stream, how to build the parse tree?*
  - bottom up parsing
  - top down parsing
- *How to make sure parse tree is unique? (the ambiguity problem)*
- *Given a CFG, how to build a parser?*  
use *ML-Yacc parser generator*
- *How to detect, report, and recover from syntax errors*

# Grammars

A *grammar* is a precise specification of a programming language *syntax*.

A grammar is normally specified using *Bachus-Naur Form* (BNF):

1. two sets of symbols

*terminal*: **if**, **id**, **(**, **)** (the lexical tokens)

*nonterminal*: *stmt*, *expr* (the phrase classes)

2. a set of *productions* or *rewriting rules*

*stmt*  $\rightarrow$  **if** *expr* **then** *stmt* **else** *stmt*

*expr*  $\rightarrow$  *expr* **+** *expr* | *expr* **\*** *expr*  
| **(** *expr* **)** | **id**

The latter abbreviates the 4 rules:

*expr*  $\rightarrow$  *expr* **+** *expr*

*expr*  $\rightarrow$  *expr* **\*** *expr*

*expr*  $\rightarrow$  **(** *expr* **)**

*expr*  $\rightarrow$  **id**

# Context-Free Grammars (CFG)

A *context-free grammar* is defined as a quadruple  $\langle T, N, P, S \rangle$ , where

$T$  is a finite set of terminal symbols

$N$  is a finite set of nonterminal symbols

$P$  is a finite set of *productions*:

$$N \rightarrow \sigma \quad \text{with } N \in N \text{ and } \sigma \in (N \cup T)^*$$

$S \in N$  is the *start symbol*

## Example

$$T = \{ +, *, (, ), \mathbf{id} \}$$

$$N = \{ E \}$$

$$P = \{ E \rightarrow E + E, E \rightarrow E * E, E \rightarrow (E), E \rightarrow \mathbf{id} \}$$

$$S = E$$

BNF:  $E \rightarrow E + E \mid E * E \mid ( E ) \mid \mathbf{id}$

# Derivations

A sentence is a string of terminal symbols (or tokens).

A derivation is a sequence of strings in  $(N \cup T)^*$ , starting with the start symbol  $S$ , where each string is produced by replacing a nonterminal with the rhs of one of its productions.

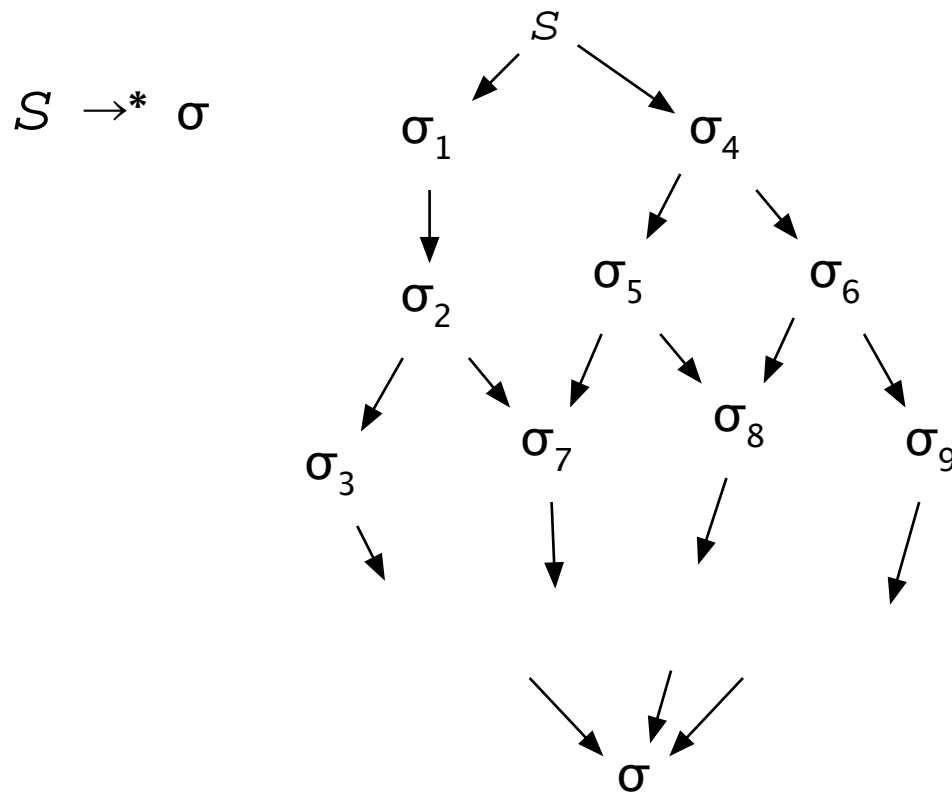
$E$   
 $E + \underline{E}$   
 $E + \underline{E} * E$   
 $\underline{E} + id * E$   
 $(E) + id * \underline{E}$   
 $(\underline{E}) + id * id$   
 $(\underline{E} * E) + id * id$   
 $(id * \underline{E}) + id * id$   
 $(id * id) + id * id$

*a sentence (no nonterminals)*

$E$   
 $E + \underline{E}$   
 $E + E * \underline{E}$   
 $E + \underline{E} * id$   
 $\underline{E} + id * id$   
 $(\underline{E}) + id * id$   
 $(E * \underline{E}) + id * id$   
 $(\underline{E} * id) + id * id$   
 $(id * id) + id * id$

*a leftmost derivation*

# Multiple Derivations



There will be multiple derivations taking the start symbol  $S$  to a terminal sentence  $\sigma$ , depending on order in which productions are applied. Each path determines a parse tree.



# Language of a CFG

A derivation is a sequence

$$S \rightarrow \sigma_1 \rightarrow \sigma_2 \rightarrow \sigma_3 \rightarrow \dots \rightarrow \sigma_n \quad (\text{or } S \rightarrow^* \sigma_n)$$

where  $\sigma_n$  consists only of terminal symbols ( $\sigma_n \in T^*$ ).

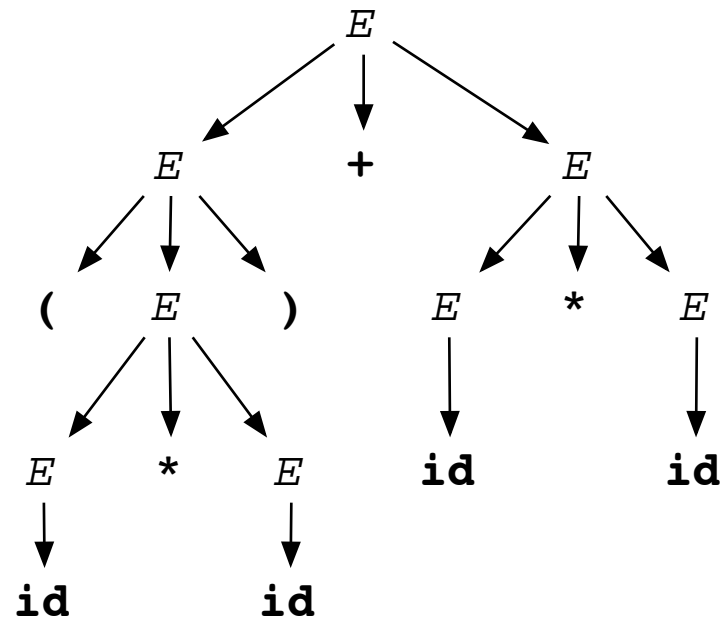
The language  $L(G)$  defined by grammar  $G = \langle T, N, P, S \rangle$  is the set of strings of terminals that are derivable from  $S$ :

$$L(G) = \{ \sigma \in T^* \mid S \rightarrow^* \sigma \}$$

# Parse Trees

A *parse tree* is a graphical representation of a derivation, but the order of nonterminal replacements is not indicated.

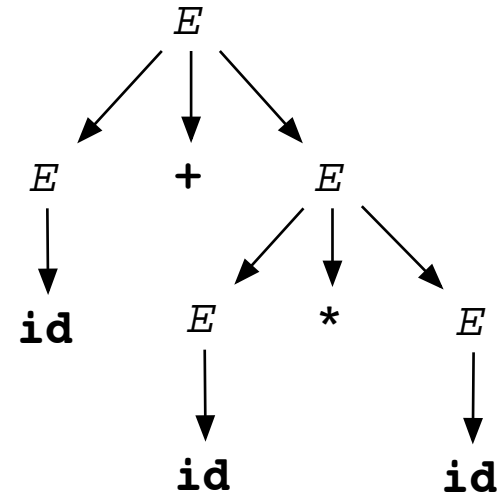
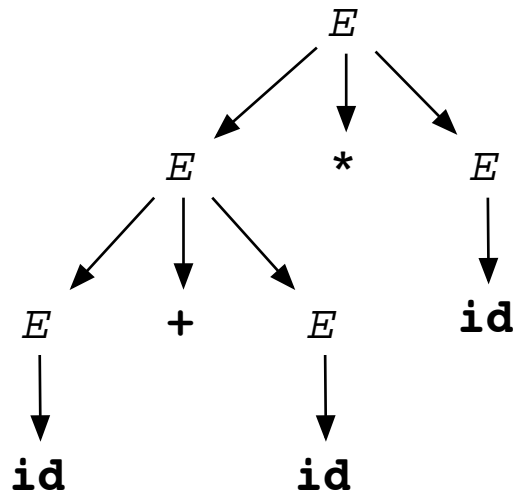
**(id \* id) + id \* id**



# Ambiguity

A single sentence can have multiple parse trees, meaning that its structure is ambiguous. We say the CFG is *ambiguous*.

**id + id \* id**



# Removing Ambiguity

There are techniques for transforming a grammar to remove certain kinds of ambiguities.

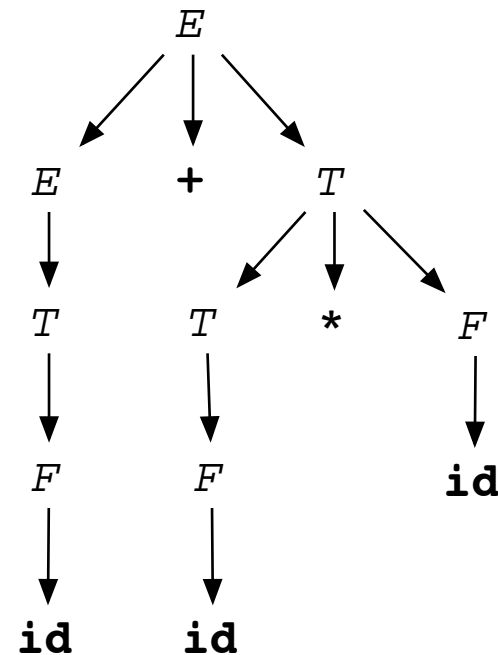
*Ambiguous*

$$\begin{aligned} E &\rightarrow E + E \\ E &\rightarrow E * E \\ E &\rightarrow (E) \\ E &\rightarrow \text{id} \end{aligned}$$

*Unambiguous*

$$\begin{aligned} E &\rightarrow E + T \\ T &\rightarrow T * F \\ F &\rightarrow (E) \\ F &\rightarrow \text{id} \end{aligned}$$

**id + id \* id**



*Idea: Express precedence through new nonterminals.*

# Top-Down parsers

A top-down parser tries to construct a parse tree top-down as it scans the token stream from left to right.

This can require *backtracking*, but most programming languages can be parsed without backtracking.

*Recursive descent* or *predictive parsing* is a type of top-down parsing that can be used when:

1) *production rules can be distinguished based on the possible first tokens of sentences derived from their rhs (FIRST sets)*

2) *there are no left-recursive productions (e.g.  $E \rightarrow E + E$  )*

# Recursive Descent

$S \rightarrow \text{if } E \text{ then } S \text{ else } S$

$L \rightarrow \text{end}$

$S \rightarrow \text{begin } S \ L$

$L \rightarrow ; \ S \ L$

$S \rightarrow \text{print } E$

$E \rightarrow \text{num} = \text{num}$

First sets:

$\text{FIRST}(\text{if } E \text{ then } S \text{ else } S) = \{\text{if}\}$

$\text{FIRST}(\text{begin } S \ L) = \{\text{begin}\}$

$\text{FIRST}(\text{print } E) = \{\text{print}\}$

$\text{FIRST}(\text{end}) = \{\text{end}\}$

$\text{FIRST}( ; \ S \ L) = \{ ; \}$

$\text{FIRST}(\text{num} = \text{num}) = \{\text{num}\}$

Note that each rule is uniquely determined by the first symbol of the sentences it generates.

# Recursive Descent Parser

```
datatype token = IF | THEN | ELSE | BEGIN | END |  
                | PRINT | SEMI | NUM | EQ
```

```
val nexttok = ref(getToken())  
fun match t = if !nexttok = t  
              then nexttok := getToken()  
              else error()
```

```
fun S() = case !nexttok  
        of IF => (match IF; E(); match THEN; S();  
                match ELSE; S())  
        | BEGIN => (match BEGIN; S(); L())  
        | PRINT => (match PRINT; E())
```

```
and L() = case !nexttok  
        of END => match END  
        | SEMI => (match SEMI; S(); L())
```

```
and E() = (match NUM; match EQ; match NUM)
```

# Recursive Descent Parser

## Notes:

There is a set of recursive functions, one representing each nonterminal symbol.

For each of these functions there is a case rule for each production for that nonterminal, guarded by the first symbol generated by the rhs of the production.

Each production has a unique first symbol that can be used to distinguish it from other possible productions. In general there might be a set of possible first symbols, but these sets would need to be disjoint for the different productions so they could be used to “predict” the proper production.



# Left Recursive Productions

A production like

$$E \rightarrow E + T$$

is bad because it would lead to a function definition for E of the form:

```
fun E() = (E(); match PLUS; T())
```

which would clearly not terminate -- it would not even look at the next token.

There is a systematic way to transform productions to eliminate left recursion. This results in:

$$E \rightarrow T R$$

$$R \rightarrow + T R$$

$$R \rightarrow \epsilon$$

# Eliminating Left Recursion

Transform a left recursive production of the form

$$\begin{aligned} A &\rightarrow A \alpha \\ A &\rightarrow \beta \end{aligned}$$

by introducing a new nonterminal  $R$  with productions

$$\begin{aligned} A &\rightarrow \beta R \\ R &\rightarrow \alpha R \\ R &\rightarrow \epsilon \end{aligned}$$

This can be generalized if there are several left recursive productions:

$$\begin{array}{l} A \rightarrow A \alpha \\ A \rightarrow A \gamma \\ A \rightarrow \beta \end{array} \quad \Rightarrow \quad \begin{array}{l} A \rightarrow \beta R \\ A \rightarrow \gamma R \\ R \rightarrow \alpha R \\ R \rightarrow \epsilon \end{array}$$

# Top-Down parsers

We need to formally define the set of possible first tokens generated from a sentence  $\alpha \in (N \cup T)^*$  (a production rhs).

$\text{FIRST}(\alpha)$  is the set of possible first tokens that can occur in sentences generated from  $\alpha$ . If the string  $\alpha$  starts with a nonterminal, then that nonterminal constitutes the FIRST set. If  $\alpha$  starts with a terminal, we may have to resort to the more complicated algorithm described in Algorithm 3.13 (Appel, p. 49).