CMSC 15300 Homework problem (Due 5/28/2003)

Consider the following definition of binary trees labeled with integers (in SML syntax):

```
datatype tree = Leaf | Nd of (tree * int * tree);
```

Values of this type include:

```
Leaf
Nd(Leaf, 1, Leaf)
Nd(Nd(Leaf, 1, Leaf), 5, Nd(Leaf, 1, Leaf))
```

We define the *values* of a tree inductively as follows:

$$\begin{aligned} \text{values}(\texttt{Leaf}) &= \emptyset \\ \text{values}(\texttt{Nd}(t_1, x, t_2)) &= \{x\} \cup \text{values}(t_1) \cup \text{values}(t_2) \end{aligned}$$

Binary search trees are trees in which an inorder traversal produces an increasing sequence of node labels. We can formalize this property with the following definition:

$$\begin{aligned} \text{BST}(t) \quad \equiv \quad & (t = \texttt{Nd}(t_1, x, t_2)) \Rightarrow \\ & \quad [\forall y (y \in \text{values}(t_1) \Rightarrow (y < x)) \wedge \forall y (y \in \text{values}(t_2) \Rightarrow (x < y))] \end{aligned}$$

We can use a binary search tree to represent sets of integers. The following SML function tests to see if its first argument is a member of the set represented by its second argument:

```
fun member (x, t) = (case t
       of Leaf => false
        | Nd(t1, y, t2) => if (x < y) then member(x, t1)
                     else if (x > y) then member(x, t2)
                                      else true
      (* end case *))
```

Prove, by induction, the following correctness statement:

$$\forall t : \texttt{tree}\ [\text{BST}(t) \Rightarrow (\texttt{member}(x, t) \Leftrightarrow x \in \text{values}(t))]$$